ENERGY EXCHANGE BETWEEN TURBULENT AND ZONAL MODES IN COLLISIONLESS TRAPPED ELECTRON MODE (CTEM) TURBULENCE

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Abstract

We use spectral data obtained by a numerical simulation of 2-field (electron density and electrostatic potential) CTEM turbulence to analyze energy exchange between turbulent and zonal modes. The analysis is based on the examination of the equation for the total turbulent energy. The only three terms originating from the ExB nonlinearity that are not annihilated by the summation over k (and that thus lead to a net transfer of energy in or out of the turbulence) are: (1) the nonlinear growth rate term; (2) a term proportional to triplet correlations containing two turbulent modes and a zonal potential mode, and (3) a term proportional to triplet correlations containing two turbulent modes and a zonal density mode. When averaged over time, the first term is positive while the last two terms are negative. The time-average over the initial run-up phase and over the saturation phase of the ratio between term (2) and term (3) is about 0.5, so that the major contribution to the suppression of turbulent energy due to zonal modes comes from the term proportional to the zonal density, and thus is not related to the shearing suppression mechanism often relevant for a mean shear flow. We analyze further term (2) to determine the relative importance of ExB shearing versus spectral transfer to the damped eigenmode as the dominant saturation mechanism.

MOTIVATION AND GOAL

- Numerical studies of drift-wave turbulence show how zonal modes are nonlinearly excited, and that their presence is correlated with a lower saturation level of the turbulence.
  Experimentally, it is seen that zonal modes can be driven by coupling of density and flow (Reynolds stress not required)
- Goal: *Study the energy exchange between turbulent ($k_y \neq 0$) and zonal ($k_y = 0$) modes in CTEM turbulence to clarify how zonal modes suppress turbulence*

MAIN FINDINGS

- The ExB nonlinearity leads to both excitation and, indirectly to, damping of zonal modes by changing the dissipation rate
- The nonlinear damping of zonal modes is active during the entire system evolution: at early times, it slows down the growth of the turbulent energy, while at saturation it sets the level of the turbulence
- The nonlinear damping of zonal modes is a turbulence suppression mechanism which is not related to shearing of turbulent eddies, the latter being often relevant in the case of mean flows

A TWO-FIELD MODEL OF CTEM TURBULENCE

Effective electron density evolution:

$$\frac{\partial n(\mathbf{x},t)}{\partial t} + \nu(n-\phi) + v_d\hat{\alpha}\frac{\partial\phi}{\partial y} = \overbrace{\boldsymbol{\nabla}\phi\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}n}^{E\times B\ \mathrm{NL}}$$

Vorticity evolution:

$$\frac{\partial}{\partial t}\left(1-\boldsymbol{\nabla}^2-\varepsilon^{1/2}\right)\phi(\mathbf{x},t) - \varepsilon^{1/2}\nu(n-\phi) \ + \ v_d(1-\varepsilon^{1/2}\hat{\alpha})\frac{\partial\phi}{\partial y}$$

$$= - \overbrace{\boldsymbol{\nabla}\phi\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2\phi}^{\text{Polar. NL: neglegible. for } \lambda\to\infty}$$

where

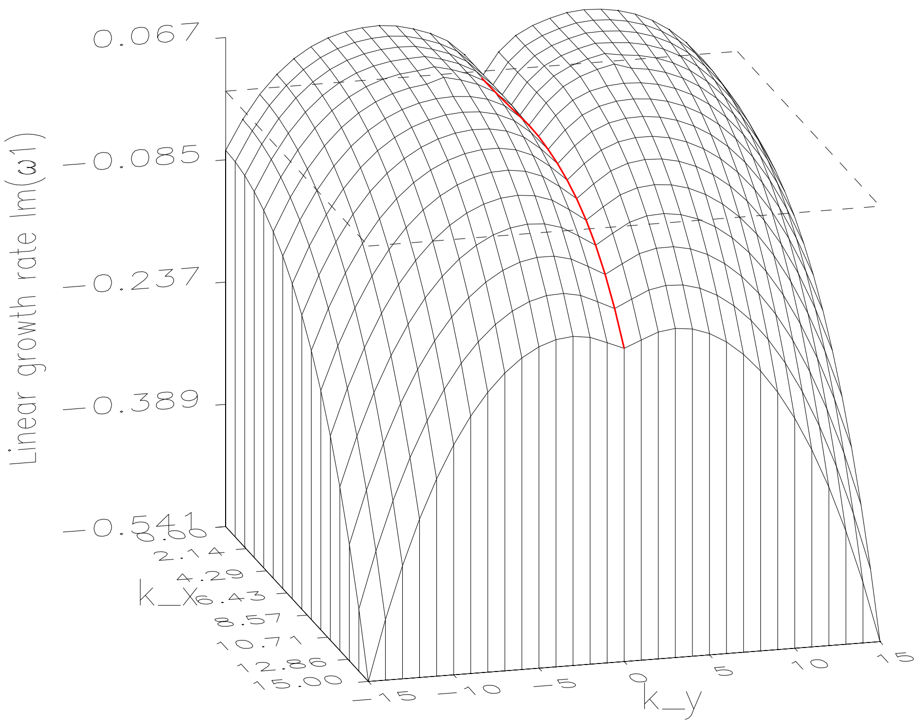
- ϕ: electrostatic potential
- $n = \phi + n_e/\varepsilon^{1/2}$: effective density ($n_e = \phi + \varepsilon^{1/2}\hat{n}_\mathrm{e}$, where $\hat{n}_\mathrm{e}$ is the trapped electron density)
- ν: trapped particle scattering, $\varepsilon^{1/2}$: trapping fraction

We will consider this model in the *collisionless* $[\delta \equiv \nu/\omega_* \ll 1]$ regime

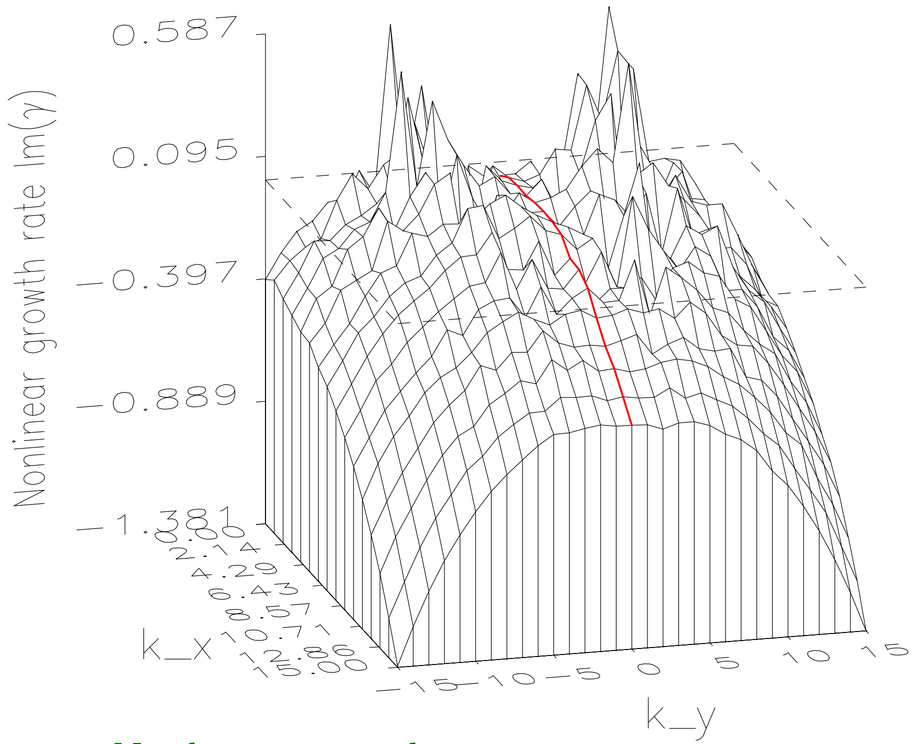
NUMERICAL RESULTS

Note: all numerical results presented in this section are obtained neglecting the polarization nonlinearity

Linear and nonlinear growth rates

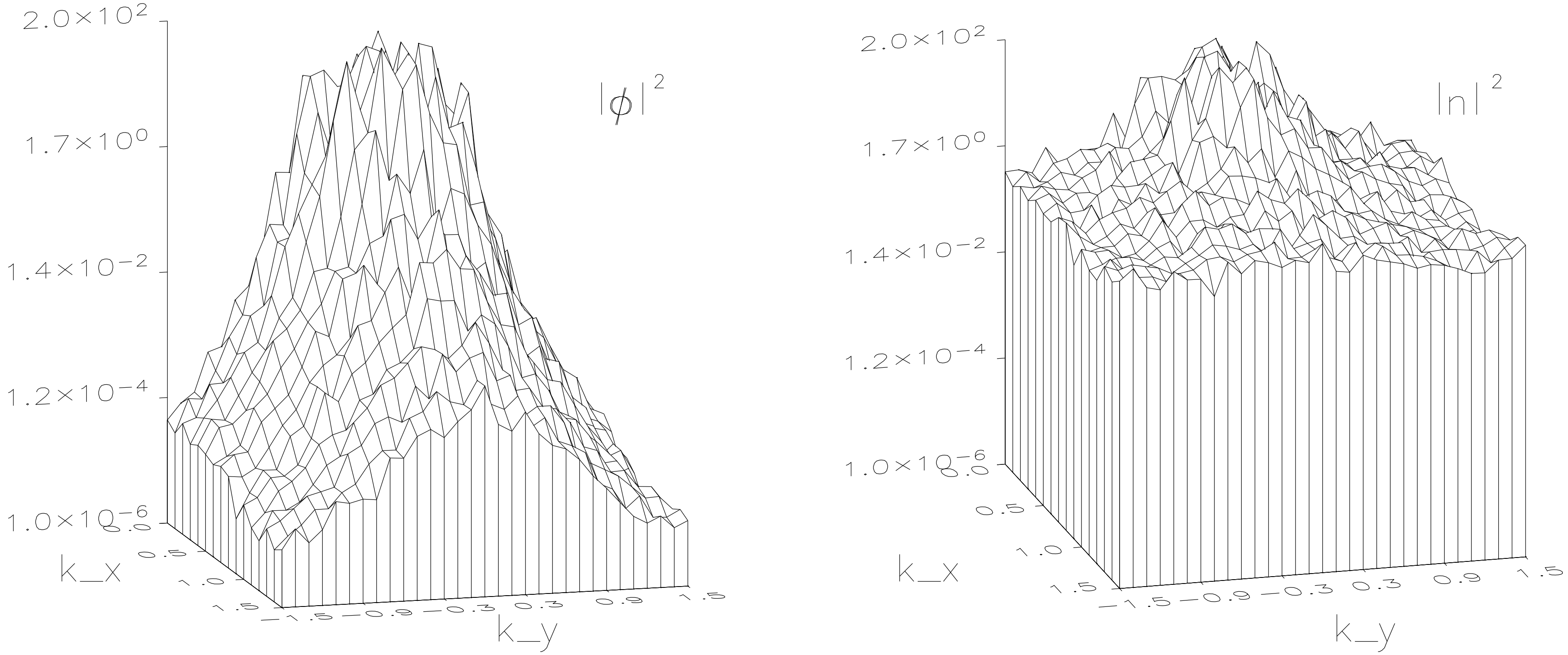


Linear growth rate (——— : zonal modes)



Nonlinear growth rate at saturation (——— : zonal modes)

Saturated levels of potential and density



At saturation, $|\phi(\mathbf{k})|^2 < |n(\mathbf{k})|^2$

Differentiation between zonal modes and "turbulent" modes

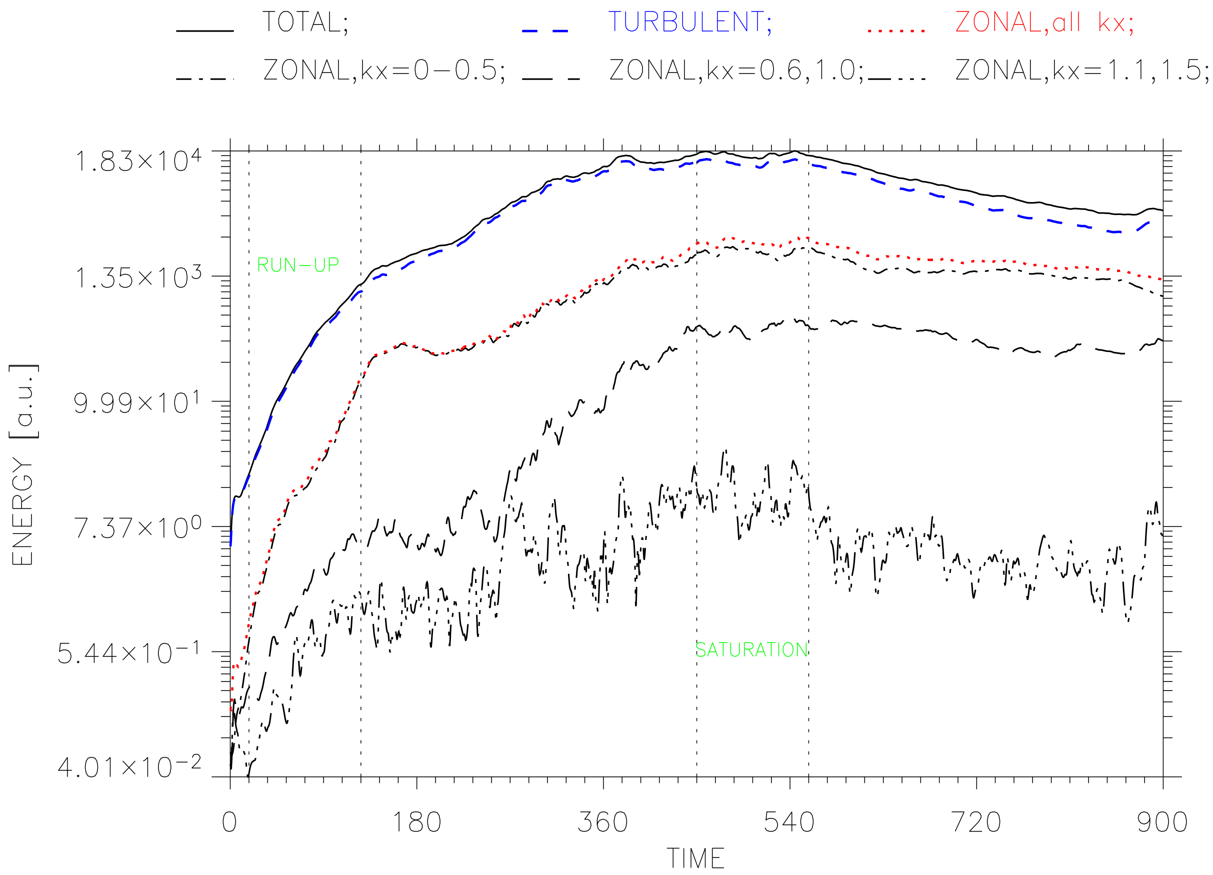
$f^{\mathrm{t}}(k_x, k_y \neq 0)$	"Turbulent modes"
$f^{\mathrm{z}}(k_x, k_y = 0)$	Zonal modes

$$E^{\mathrm{t}} \equiv \sum_{k_x, k_y \neq 0} E(k_x, k_y)$$ "Total turbulent energy" : energy in all modes with $k_y \neq 0$

$$E^{\mathrm{z}} \equiv \sum_{k_x} E(k_x, k_y = 0)$$ Zonal mode energy : energy in all modes with $k_y = 0$

Note: zonal modes are part of the turbulent spectrum, too!

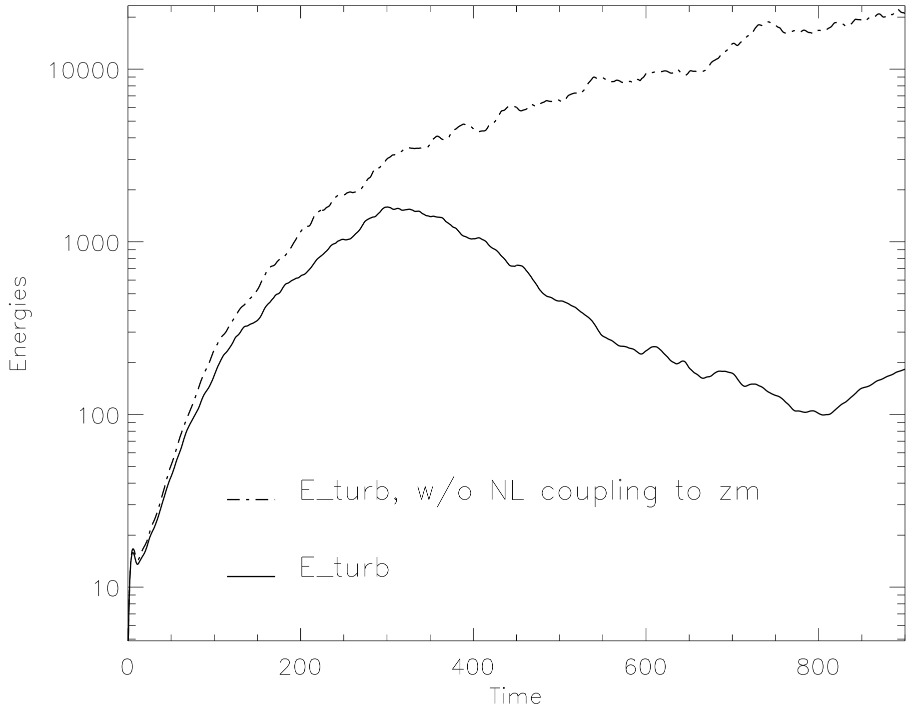
Time evolution of relevant energies ($\nu = 0.1$).



Linearly damped zonal modes are nonlinearly excited to finite amplitude

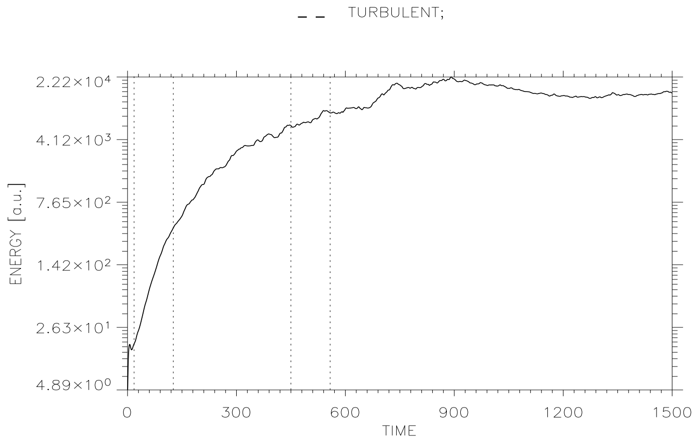
Vertical lines mark the initial run-up interval $t = [19.8, 126]$, and the saturation interval $t = [452, 558]$.

Total turbulent energy with (———) and without ($- \cdot - \cdot -$) nonlinear interaction with zonal modes ($\nu = 0.05$)



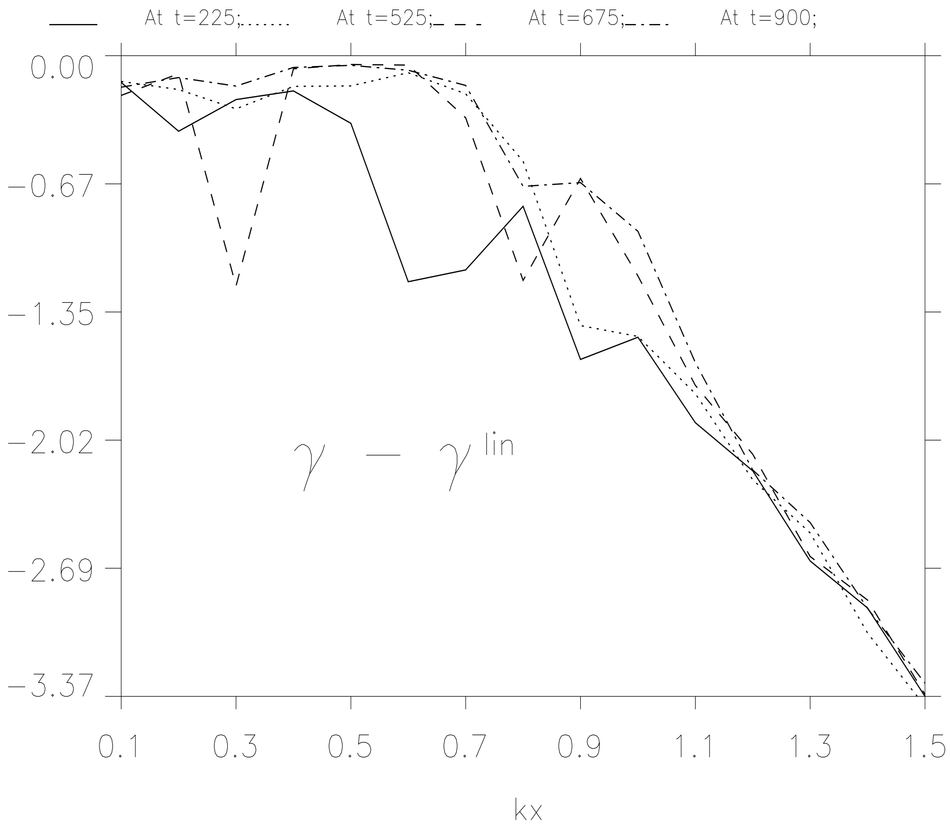
Turbulent energy evolves to a much higher amplitude when nonlinear interaction with zonal modes is artificially removed

However, *turbulence saturates even without zonal mode coupling* (it takes longer, and the saturation level is higher):



Finite-amplitude (nonlinear) damping of zonal modes

Quantity $\underline{\gamma(k_x, k_y = 0) - \gamma^{\rm lin}(k_x, k_y = 0)}$ at four times during the simulation:



$\gamma(k_x, k_y = 0) - \gamma^{\rm lin}(k_x, k_y = 0) < 0$

$\Rightarrow$ zonal modes are nonlinearly damped for all k_x at any instant of the simulation

STUDY OF SPECTRAL ENERGY TRANSFER

Note: all numerical results presented in this section are obtained neglecting the polarization nonlinearity

Equation for the energy in mode $\mathbf{k} = (k_x, k_y)$

$$\boxed{\frac{1}{2}\frac{dE(\mathbf{k})}{dt} = \gamma(\mathbf{k})E(\mathbf{k}) + \frac{1}{2}\mathcal{T}(\mathbf{k})}$$

where the energy is

$$E(\mathbf{k}) = (1 + k^2 - \varepsilon^{1/2})|\phi(\mathbf{k})|^2 + \varepsilon^{1/2}|n(\mathbf{k})|^2 \ ,$$

the growth rate is

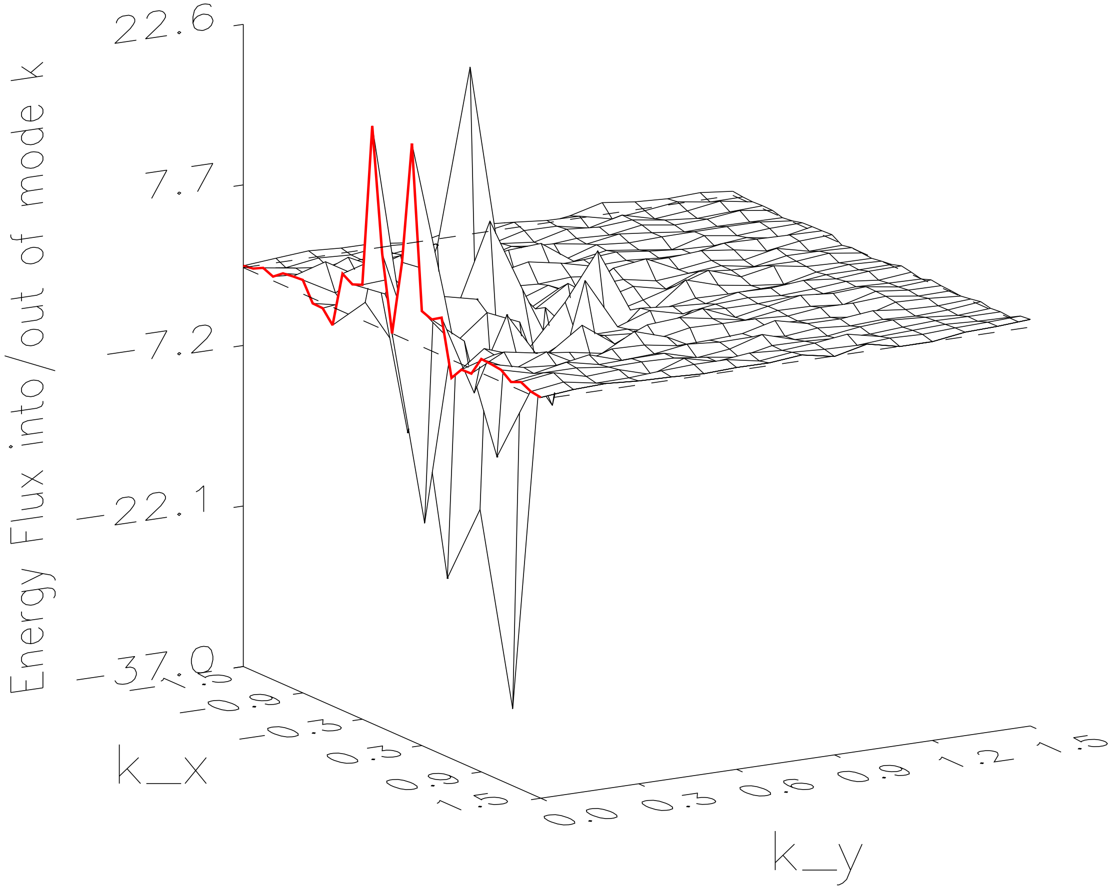
$$\gamma(\mathbf{k})E(\mathbf{k}) = k_y v_{\mathrm{d}}\hat{\alpha}\varepsilon^{1/2}\Im\langle\phi(\mathbf{k})n(-\mathbf{k})\rangle - \nu\varepsilon^{1/2}\left|n(\mathbf{k}) - \phi(\mathbf{k})\right|^2 \ ,$$

and the nonlinear spectral energy transfer is

$$\mathcal{T}(\mathbf{k}) = -\sum_{\mathbf{k}'}\varepsilon^{1/2}(k_x k'_y - k_y k'_x)\Re\left[\langle n(-\mathbf{k})\phi(\mathbf{k}')n(\mathbf{k}-\mathbf{k}')\rangle - \langle n(-\mathbf{k})\phi(\mathbf{k}-\mathbf{k}')n(\mathbf{k}')\rangle\right]$$

Spectral energy transfer

Quantity $\underline{\mathcal{T}(\mathbf{k})}$ giving the energy flux into (positive values) or out of (negative values) the generic mode $\mathbf{k}$ due to interactions with all remaining modes in the spectrum, and time-averaged over the entire saturation interval



Energy flux from turbulence to zonal modes (——) is positive (into zonal modes) and large.

Equations for total energies

$$\boxed{\begin{aligned}\frac{1}{2}\frac{dE^{\mathrm{t}}(t)}{dt} &= \gamma^{\mathrm{t}}E^{\mathrm{t}} + \frac{1}{2}\left(\mathcal{T}_\phi + \mathcal{T}_{n_\mathrm{e}}\right)\\ \frac{1}{2}\frac{dE^{\mathrm{z}}(t)}{dt} &= \gamma^{\mathrm{z}}E^{\mathrm{z}} - \frac{1}{2}\left(\mathcal{T}_\phi + \mathcal{T}_{n_\mathrm{e}}\right)\end{aligned}}$$

where the growth rate terms are (in terms of ϕ and n_e)

$$\gamma^{\mathrm{t}}E^{\mathrm{t}} = \sum_{k_x, k_y \neq 0}\left[k_y v_\mathrm{d}\hat{\alpha}\Im\langle n_\mathrm{e}^{\mathrm{t}}(-k_x,-k_y)\phi^{\mathrm{t}}(k_x,k_y)\rangle + k_y v_\mathrm{d}\hat{\alpha}\varepsilon^{1/2}\left|\phi^{\mathrm{t}}(k_x,k_y)\right|^2 - \frac{\nu}{\varepsilon^{1/2}}\left|n_\mathrm{e}^{\mathrm{t}}(k_x,k_y)\right|^2\right]$$

$$\gamma^{\mathrm{z}}E^{\mathrm{z}} = -\sum_{k_x}\frac{\nu}{\varepsilon^{1/2}}\left|n_\mathrm{e}^{\mathrm{z}}(k_x,0)\right|^2$$

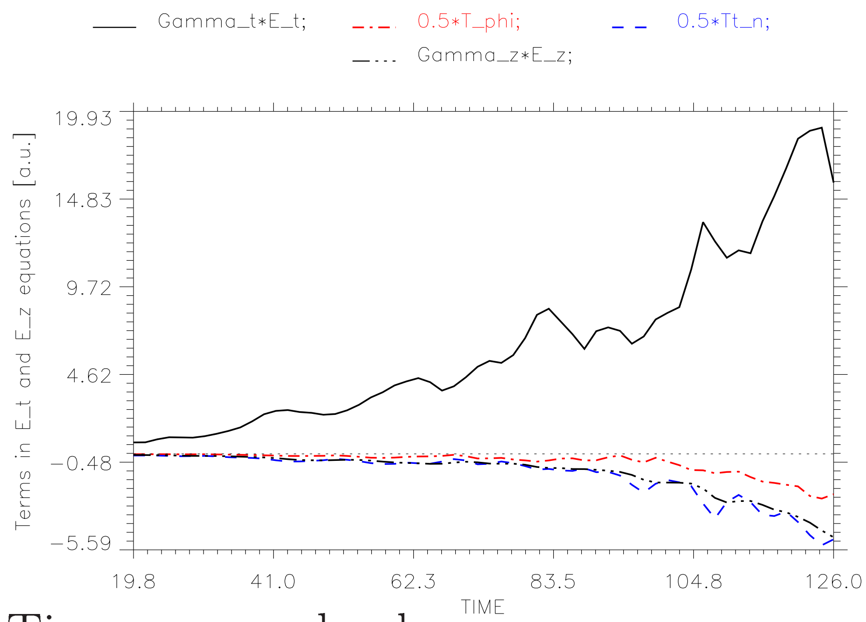
and the nonlinear spectral energy transfer terms [1] are

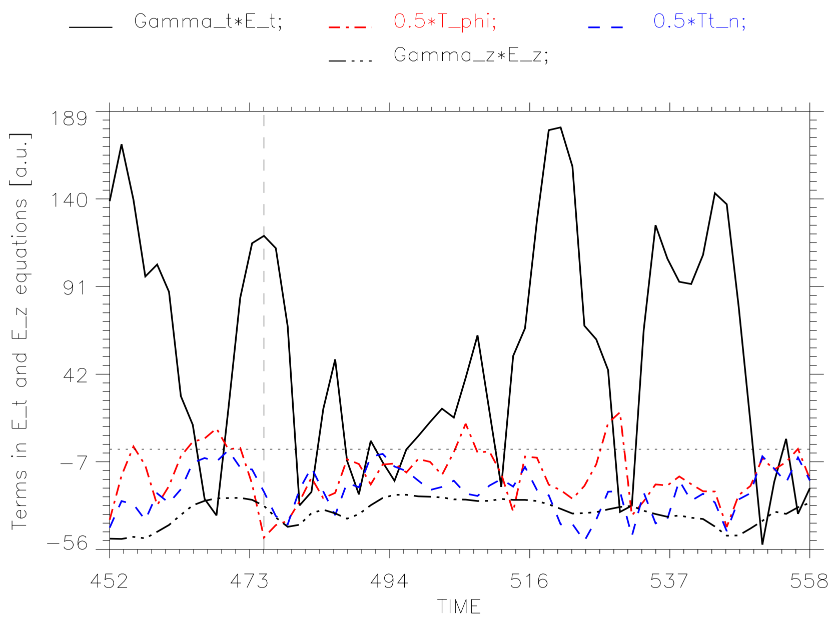
$$\mathcal{T}_\phi = \sum_{k_x, k_y \neq 0}\mathcal{T}_\phi(k_x,k_y) = -2\Re\sum_{k_x, k_y \neq 0, k'_x} k_y k'_x\,\langle n_\mathrm{e}^{\mathrm{t}}(-k_x,-k_y)\phi^{\mathrm{t}}(k_x-k'_x,k_y)\phi^{\mathrm{z}}(k'_x,0)\rangle$$

$$\mathcal{T}_{n_\mathrm{e}} = \sum_{k_x, k_y \neq 0}\mathcal{T}_{n_\mathrm{e}}(k_x,k_y) = -2\Re\sum_{k_x, k_y \neq 0, k'_x}\frac{k_y k'_x}{\varepsilon^{1/2}}\langle n_\mathrm{e}^{\mathrm{t}}(-k_x,-k_y)\phi^{\mathrm{t}}(k_x-k'_x,k_y)n_\mathrm{e}^{\mathrm{z}}(k'_x,0)\rangle$$

[1] These succinct expressions are valid only if the k'_x sums are performed over the entire spectrum

Time evolution of the equation's terms $\underline{\gamma^{\mathrm{t}} E^{\mathrm{t}}}$, $\underline{\mathcal{T}_\phi}$, $\underline{\mathcal{T}_{n_\mathrm{e}}}$ and $\underline{\gamma^{\mathrm{z}} E^{\mathrm{z}}}$ during the <u>initial run-up</u> and <u>saturation</u> phases





Time-averaged values:

$\gamma^{\mathrm{t}} E^{\mathrm{t}} = +6.38 \quad \gamma^{\mathrm{z}} E^{\mathrm{z}} = -1.12$

$\gamma_t E_t = +51.34 \quad \gamma_z E_z = -34.94$

$\mathcal{T}_\phi/2 = -0.49 \quad \mathcal{T}_{n_\mathrm{e}}/2 = -1.25$

$\mathcal{T}_\phi/2 = -14.39 \quad \mathcal{T}_{n_\mathrm{e}}/2 = -23.39$

- $\mathcal{T} < 0 \;\Rightarrow$ transfer into zonal modes
- $\mathcal{T}_\phi < \mathcal{T}_{n_\mathrm{e}} \Rightarrow$ shearing of turbulence is not the dominant suppression mechanism
- $\dot{E}^{\mathrm{z}}/E^{\mathrm{z}}\big|_{\mathrm{run-up}} = +0.0193 > \dot{E}^{\mathrm{t}}/E^{\mathrm{t}}\Big|_{\mathrm{run-up}} = +0.0147 \quad \Rightarrow$ energy growth rate higher for zonal modes despite their nonlinear damping

Suppression of turbulent energy by zonal modes

The following picture of spectral energetics in CTEM turbulence emerges:

- Linearly unstable turbulent modes grow extracting free energy from gradients of mean quantities
- As they grow, linearly and, later on, nonlinearly unstable turbulent modes nonlinearly excite zonal modes
- The energy flow to zonal modes is mediated by triplets containing both potential and density zonal modes, the latter giving the dominant contribution:

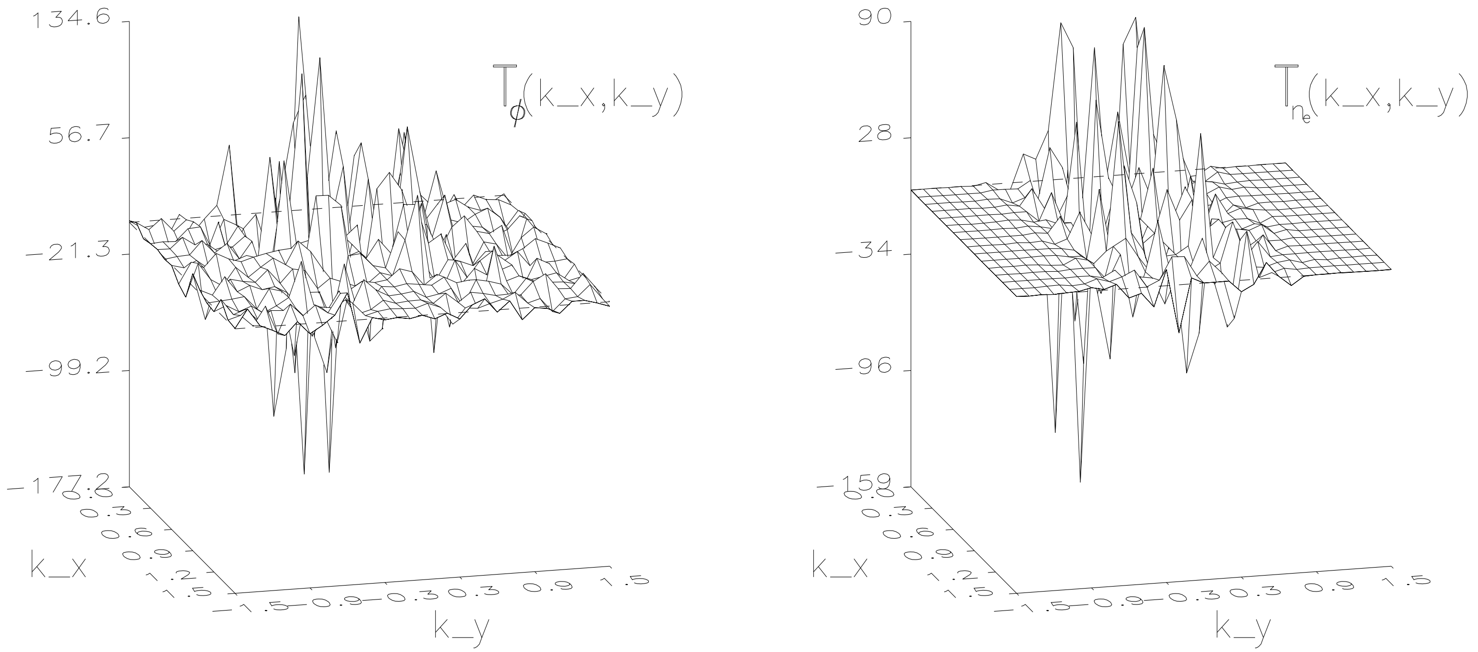
$$\mathcal{T}_{n_\mathrm{e}}/\mathcal{T}_\phi|_{\mathrm{run-up}} = 2.52 \gg 1 \ , \qquad \mathcal{T}_{n_\mathrm{e}}/\mathcal{T}_\phi|_{\mathrm{satur}} = 1.62 > 1$$

- The finite-amplitude damping of zonal modes becomes more and more important as the amplitude of the zonal modes grow, and ultimately sets the saturation level of the turbulence

The saturation levels of turbulent and zonal modes are regulated by the balance between the instability drive of the turbulent modes and the nonlinear damping of the zonal modes. This balance is reached through a net flux of energy from turbulent to zonal modes which is mostly unrelated to shearing suppression

Energy flow in/out of turbulent modes

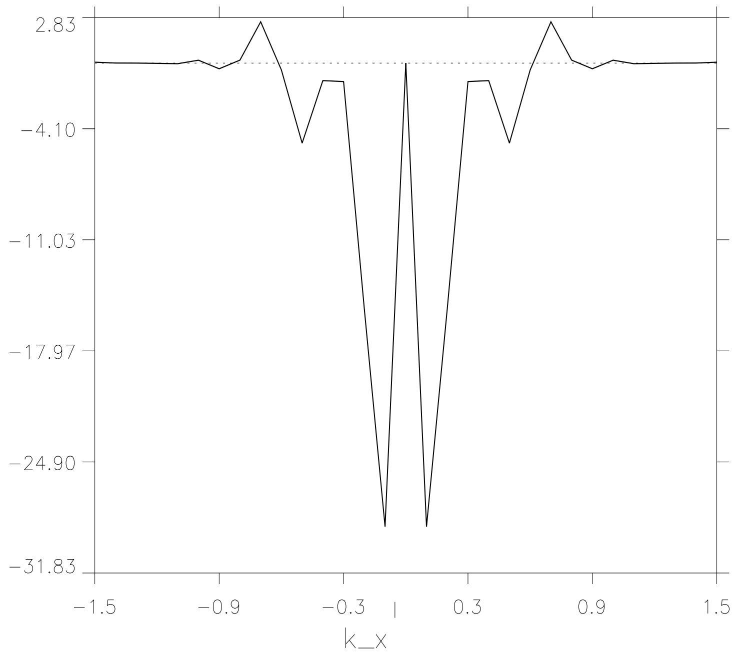
$\underline{\mathcal{T}_\phi(k_x, k_y) \propto \phi^{\mathrm{z}}}$ and $\underline{\mathcal{T}_{n_\mathrm{e}}(k_x, k_y) \propto n_\mathrm{e}^{\mathrm{z}}}$ at $t = 475.2$ in the the saturation phase. It gives the rate of energy exchange $\underline{\text{mediated by } \phi^{\mathrm{z}}}$ and $\underline{n_\mathrm{e}^{\mathrm{z}}}$ between each single turbulent mode $k_x, k_y \neq 0$ and the complete spectrum of the zonal potentials and densities. Positive (negative) peaks represent energy flow into (out of) the corresponding turbulent mode



Each single turbulent mode can receive or loose a significant amount of energy by nonlinearly interacting with zonal modes, even though the net energy flow is negative (into zonal modes)

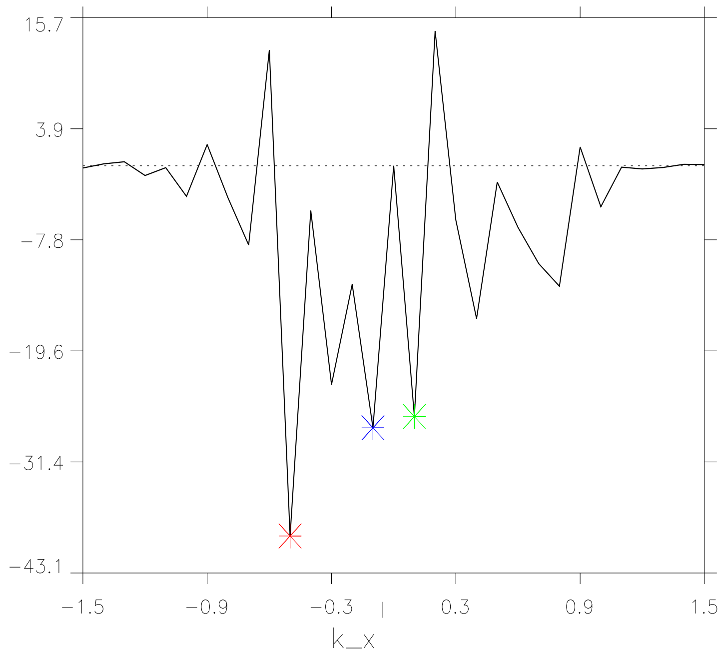
Which $\phi^{z}(k'_x)$ contributes most to suppressing turbulence?

$\underline{\mathcal{T}_\phi(k'_x) = \sum_{\mathbf{k}} \mathcal{T}_\phi(\mathbf{k}; k'_x)}$:



Zonal potentials $\phi^{z}(k'_x)$ with $k_x = [-0.5, +0.5]$ contribute most to the suppression of the total turbulent energy (at $t = 475.2$ in the saturation interval)

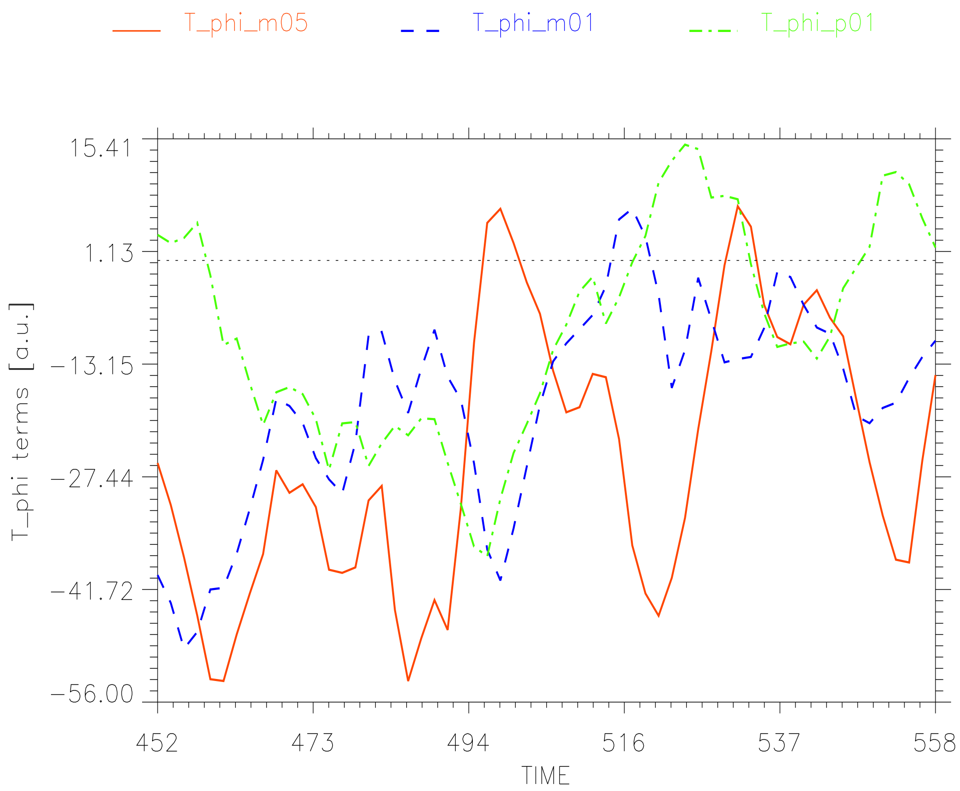
$\underline{\mathcal{T}_\phi(k_x = 0, k_y = +0.1; k'_x)}$:



Zonal potentials $\phi^{z}(k'_x)$ with $k'_x = [-0.5, +0.2]$ extract most of the energy from the specific turbulent mode $k_x = 0, k_y = +0.1$ (at $t = 475.2$ in the saturation interval)

What's the temporal behavior of the suppression during the saturation interval?

$\mathcal{T}_\phi(k_x = 0, k_y = +0.1, k'_x = -0.5)$, $\mathcal{T}_\phi(k_x = 0, k_y = +0.1, k'_x = -0.1)$ and $\mathcal{T}_\phi(k_x = 0, k_y = +0.1, k'_x = +0.1)$ as function of time during the saturation interval:



Zonal potentials $\phi^z(k'_x)$ with $k'_x = -0.5, \ -0.1$ and $+0.1$ extract energy from the specific turbulent mode $k_x = 0, k_y = +0.1$ during most, but not all, of the saturation interval

Locality of energy transfer to zonal modes

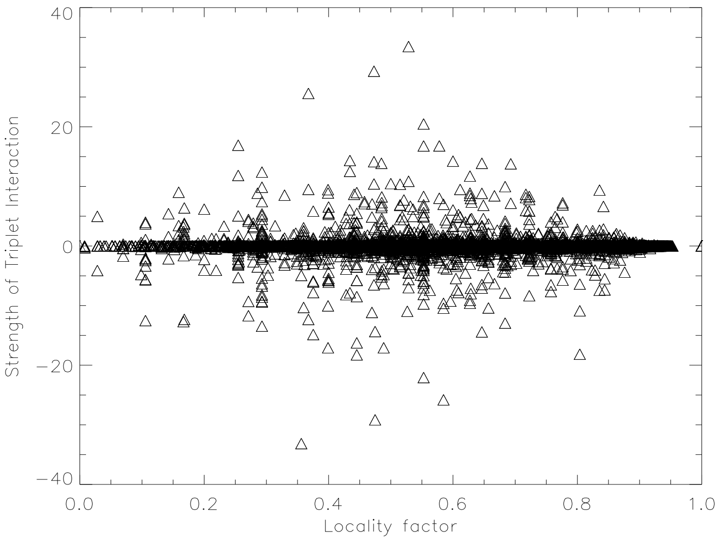
A measure of the k-space locality of the triplet interaction

$$t_\phi(k_x, k_y; k'_x) \equiv \langle n_e^t(-k_x, -k_y)\phi^t(k_x - k'_x, k_y)\phi^z(k'_x, 0)\rangle_t$$

involving the wavevectors $\mathbf{k}_1 = (-k_x, -k_y)$, $\mathbf{k}_2 = (k_x - k'_x, k_y)$ and $\mathbf{k}_3 = (k'_x, 0)$ can be given in terms of the quantity [2]

$$D_\Delta \equiv \{\mathrm{Max}[|\mathbf{k}_1|, |\mathbf{k}_2|, |\mathbf{k}_3|] - \mathrm{Min}[|\mathbf{k}_1|, |\mathbf{k}_2|, |\mathbf{k}_3|]\}/\mathrm{Max}[|\mathbf{k}_1|, |\mathbf{k}_2|, |\mathbf{k}_3|] \ .$$

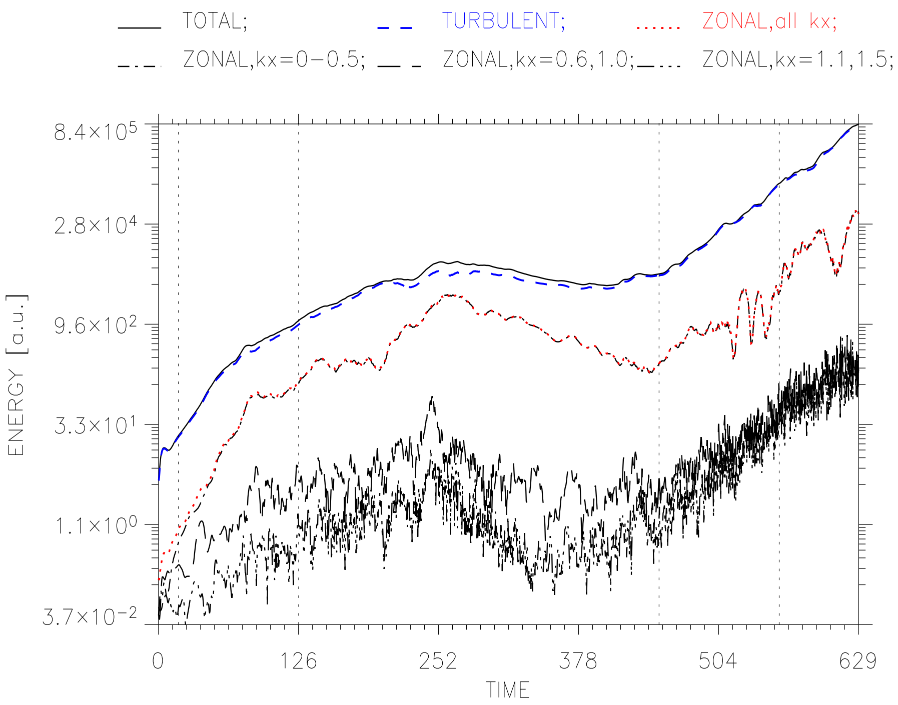
Strength of the triplet interaction $\underline{\mathcal{Q} \equiv -\Re\, k_y k'_x\ t_\phi(k_x, k_y; k'_x)}$ versus the "locality factor" $\underline{D_\Delta}$ (at the selected time $t = 475.2$ of the saturation interval):

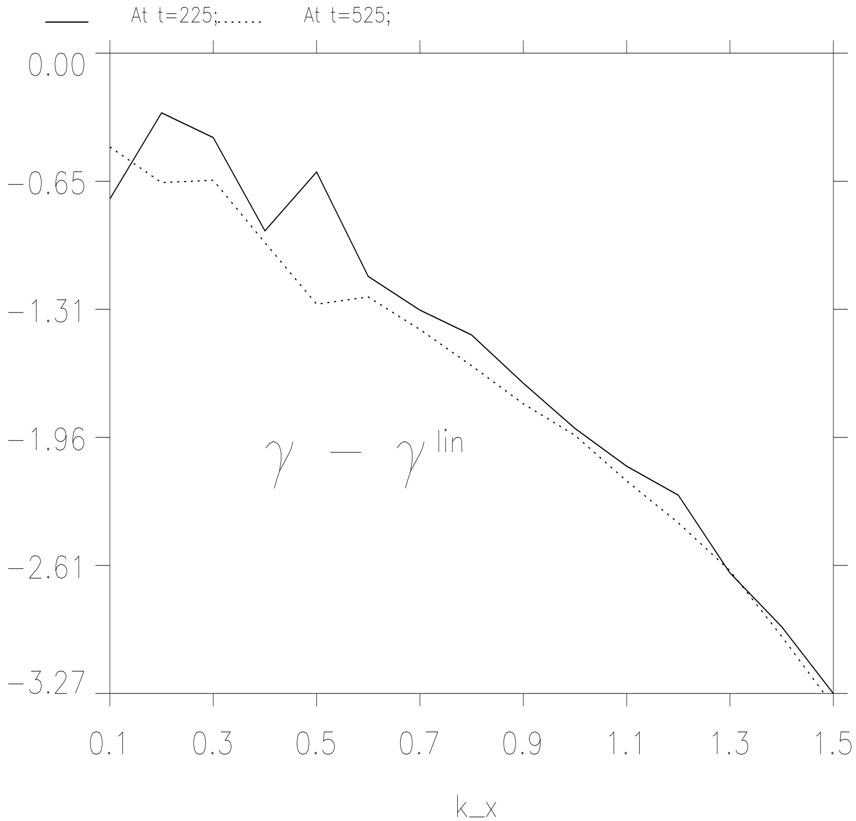


Local interaction are carrying most of the energy into zonal modes, even though nonlocal interactions ($D_\Delta \geq 0.8$) are still significant

[2] $D_\Delta = 0$ means equilateral triangle, $D_\Delta = 0.5$ means $\mathrm{Max}(\mathbf{k}) = 2 \times \mathrm{Min}(\mathbf{k})$; $D_\Delta = 0.8$ means $\mathrm{Max}(\mathbf{k}) = 5 \times \mathrm{Min}(\mathbf{k})$; $D_\Delta \to 1$ as $\mathrm{Max}(\mathbf{k})/\mathrm{Min}(\mathbf{k}) \to \infty$; the $(D_\Delta = 1, \mathcal{Q} = 0)$ point is the "zero" triangle.

EFFECT OF POLARIZATION NONLINEARITY





Nonlinear excitation and damping of zonal modes occur also when both ExB and polarization nonlinearities are active.

These results confirm that the polarization nonlinearity doesn't change qualitatively the dynamics of long-wavelength (in particular, zonal) modes

CONCLUSIONS

- In CTEM turbulence, the $E \times B$ nonlinearity, which dominates the long-wavelength dynamics, produces both excitation and, indirectly, damping of zonal modes
- *During the initial run-up*, the drive coming form turbulent modes is much stronger than the damping of zonal modes: turbulent energy grows due to the instability drive, and zonal mode energy grows due to energy transfer from turbulent modes
- *Later on*, as the amplitudes of the zonal modes become larger, the nonlinear damping of zonal modes becomes increasingly more effective in contrasting the instability drive
- *At saturation*, the amplitudes of both turbulent and zonal modes are regulated by these two competing processes: instability drive and zonal mode damping
- During the entire system evolution, there is a net flux of energy from turbulent to zonal modes which is carried by triplet interactions involving zonal potentials or zonal densities. The latter, however, give the dominant contribution to the energy transfer
- Shearing of turbulent eddies by zonal modes, if present, is not the dominant effect in lowering the saturation energy of the turbulence

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