A COMPARISON OF NONLINEAR DAMPING AND SHEARING IN ZONAL FLOW REDUCTION OF TURBULENCE [1]

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INTRODUCTION

ZONAL MODES are $k_\phi = 0, k_\theta = 0$ *fluctuations driven by an inverse energy transfer*

Numerical simulations:

- Zonal flows associated with *reduction of turbulence and transport*
- Usually attributed to the *shearing of turbulence* by the zonal flow

Experiments:

- Zonal modes observed in bicoherence measurements are stable *geodesic acoustic modes*
- In TEXT, zonal flows are driven by *coupling of density and flow* (Reynolds stress not required)

Zonal flows access nonlinear damping

Which sets saturation: shearing? nonlinear damping?

Study 2D collisionless trapped electron mode (CTEM) turbulence.

We find that

- Nonlinear growth of zonal flow modes saturates through a *strong finite-amplitude-induced damping*
- Excitation of zonal flows opens a significant *energy sink* which plays a major role to saturation of unstable turbulent fluctuations
- For CTEM, *nonlinear damping controls saturation, not shearing*

A MODEL OF COLLISIONLESS TRAPPED ELECTRON TURBULENCE

Trapped electron density evolution:

$$\frac{\partial n(\mathbf{x},t)}{\partial t} + \nu(n-\phi) + v_d\hat{\alpha}\frac{\partial \phi}{\partial y} = \boldsymbol{\nabla}\phi\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}n$$

Vorticity evolution:

$$\frac{\partial}{\partial t}\left(1-\boldsymbol{\nabla}^2-\varepsilon^{1/2}\right)\phi(\mathbf{x},t) \; - \; \varepsilon^{1/2}\nu(n-\phi)$$
$$+v_d(1-\varepsilon^{1/2}\hat{\alpha})\frac{\partial \phi}{\partial y} = -\boldsymbol{\nabla}\phi\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2\phi$$

ν: trapped particle scattering

$\varepsilon^{1/2}$: trapping fraction

The model describes both *collisionless* $[\delta \equiv \nu/\omega_* \ll 1]$ and *collisional* $(\delta \simeq 1)$ TEM instability

Nonlinear terms

- $\nabla\phi\times\hat{\mathbf{z}}\cdot\nabla n$: $E\times B$ NL term $\Rightarrow$ energy cascade to small scales. However, anisotropy of wave propagation ($\omega = k_y V_D$) induces inverse transfer to large scale zonal modes
- $\nabla\phi\times\hat{\mathbf{z}}\cdot\nabla\nabla^2\phi$: Polarization NL term $\Rightarrow$ classic inverse energy cascade to large scales
- For long wavelength turbulence

$$\nabla\phi\times\hat{\mathbf{z}}\cdot\nabla\nabla^2\phi \ll \nabla\phi\times\hat{\mathbf{z}}\cdot\nabla n$$

For analytic theory: neglect $\nabla\phi\times\hat{\mathbf{z}}\cdot\nabla\nabla^2\phi$ nonlinearity (simulations confirm unimportance of polarization at long wavelengths)

NONLINEAR GROWTH/DAMPING IN CTEM

CTEM has nonlinear damping because turbulent fluctuations not given by linear eigenmodes: *Nonlinear eigenmode* $\Rightarrow$ *nonlinear damping*

CTEM model:

$$\begin{cases} \dfrac{\partial n(\mathbf{k})}{\partial t} + \mathcal{F}_1[n(\mathbf{k}), \phi(\mathbf{k})] = 0 \\ \dfrac{\partial \phi(\mathbf{k})}{\partial t} + \mathcal{F}_2[n(\mathbf{k}), \phi(\mathbf{k})] = 0 \end{cases}$$

Linear physics

- unstable branch: eigenfrequency: $\omega = \omega_1(\mathbf{k})$ with $\Im\omega_1(\mathbf{k}) > 0$ for some $\mathbf{k}$; eigenmode: $n_1(\mathbf{k}) \propto \mathcal{O}(1)\ \phi_1(\mathbf{k})$
- stable branch: eigenfrequency: $\omega = \omega_2(\mathbf{k})$ with $\Im\omega_2(\mathbf{k}) < 0$ for all $\mathbf{k}$; eigenmode: $n_2(\mathbf{k}) \propto \mathcal{O}(1/\nu)\ \phi_2(\mathbf{k})$ $[n_2 \gg \phi_2]$

CTEM turbulence does not satisfy linear eigenmode: $n(\mathbf{k}) \neq \mathcal{O}(1)\ \phi(\mathbf{k})$

$\Rightarrow$ *it has nonlinear eigenmode structure* $\Rightarrow$ *nonlinear growth or damping*

General approach: turbulence can be decomposed into linear eigenmodes

$$\begin{pmatrix} n(\mathbf{k}) \\ \phi(\mathbf{k}) \end{pmatrix} = \beta_1 \begin{pmatrix} \mathcal{O}(1) \\ 1 \end{pmatrix} + \beta_2 \begin{pmatrix} \mathcal{O}(1/\nu) \\ 1 \end{pmatrix}$$

where the magnitude of β_2 depends on how strongly the damped eigenmode is excited.

- If $n(\mathbf{k}) \simeq \mathcal{O}(1)\ \phi(\mathbf{k}) \Rightarrow$ energy flows into/out of turbulence at linear growth/damping rate, while nonlinearities transfer energy on unstable branch from $\gamma > 0$ to $\gamma < 0$ modes
- If $n(\mathbf{k}) \neq \mathcal{O}(1)\ \phi(\mathbf{k}) \Rightarrow$ nonlinear growth or damping (CTEM)

Nonlinear growing/damping can be calculated from correlation $\Im\langle n^*_{\mathbf{k}}\phi_{\mathbf{k}}\rangle$

Energy in mode $\mathbf{k}$,

$$E(\mathbf{k}) = (1 + k^2 - \varepsilon^{1/2})|\phi_{\mathbf{k}}|^2 + \varepsilon^{1/2}|n_{\mathbf{k}}|^2 \ ,$$

evolves according to

$$\frac{dE(\mathbf{k})}{dt} = 2\gamma_{\mathbf{k}}E(\mathbf{k}) + T(\mathbf{k}) \ ,$$

where T is the nonlinear spectral transfer (conservative: $\sum_{\mathbf{k}} T(\mathbf{k}) = 0$), and $2\gamma E$ is the energy input rate:

$$\gamma_{\mathbf{k}}E(\mathbf{k}) = k_y v_d \hat{\alpha}\varepsilon^{1/2}\Im\langle n^*_{\mathbf{k}}\phi_{\mathbf{k}}\rangle - \nu\varepsilon^{1/2}|n_{\mathbf{k}} - \phi_{\mathbf{k}}|^2$$

- If $n(\mathbf{k}) \simeq \mathcal{O}(1)\ \phi(\mathbf{k}) \Rightarrow \gamma E \longrightarrow \gamma_{\rm lin}E(k)$
- If $n(\mathbf{k}) \neq \mathcal{O}(1)\ \phi(\mathbf{k}) \Rightarrow \gamma E \rightarrow \gamma_{\mathbf{k}}E(k) \neq \gamma_{\rm lin}E(k)$

CTEM: nonlinear excitation and damping of zonal flows

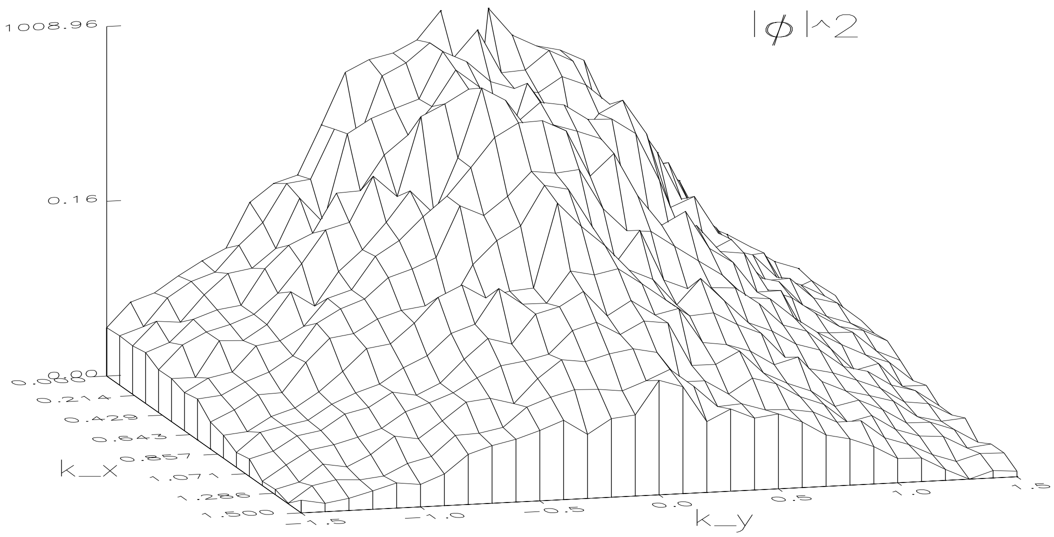
In CTEM, the $E \times B$ nonlinearity, $\boldsymbol{\nabla}\phi \times \hat{\mathbf{z}} \cdot \boldsymbol{\nabla} n$, has two effects:

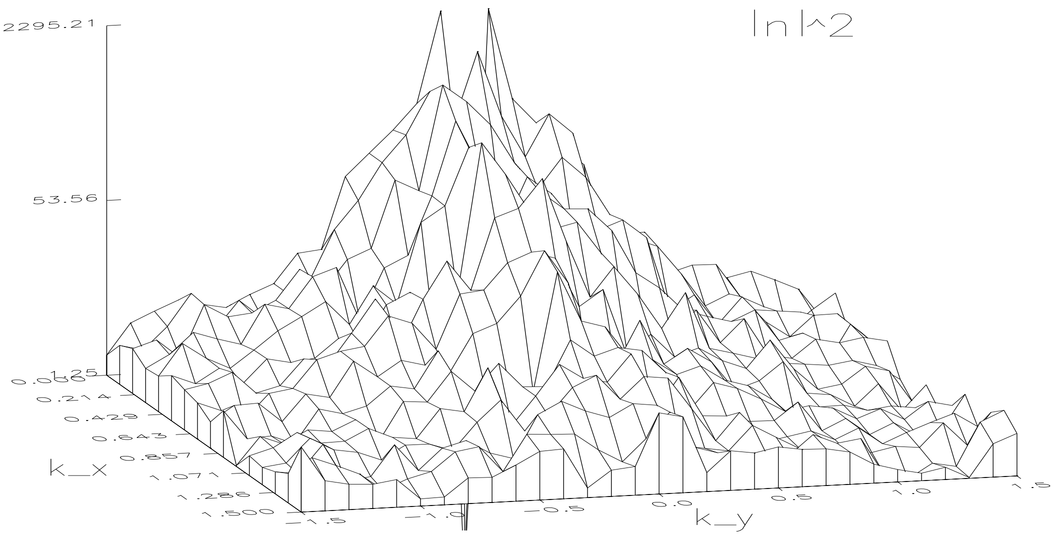
- Produce nonlinear eigenmode, i.e., $n(\mathbf{k}) > \phi(\mathbf{k})$
- drives zonal flows (as seen in simulations and experiments)
- damps zonal flows: for $k_y = 0$, $\gamma(k_y = 0)E(k_y = 0) < 0$

In CTEM, the physics that produces zonal flows also produces nonlinear damping

NUMERICAL RESULTS

At saturation: $n_{\mathbf{k}} > \phi_{\mathbf{k}}$



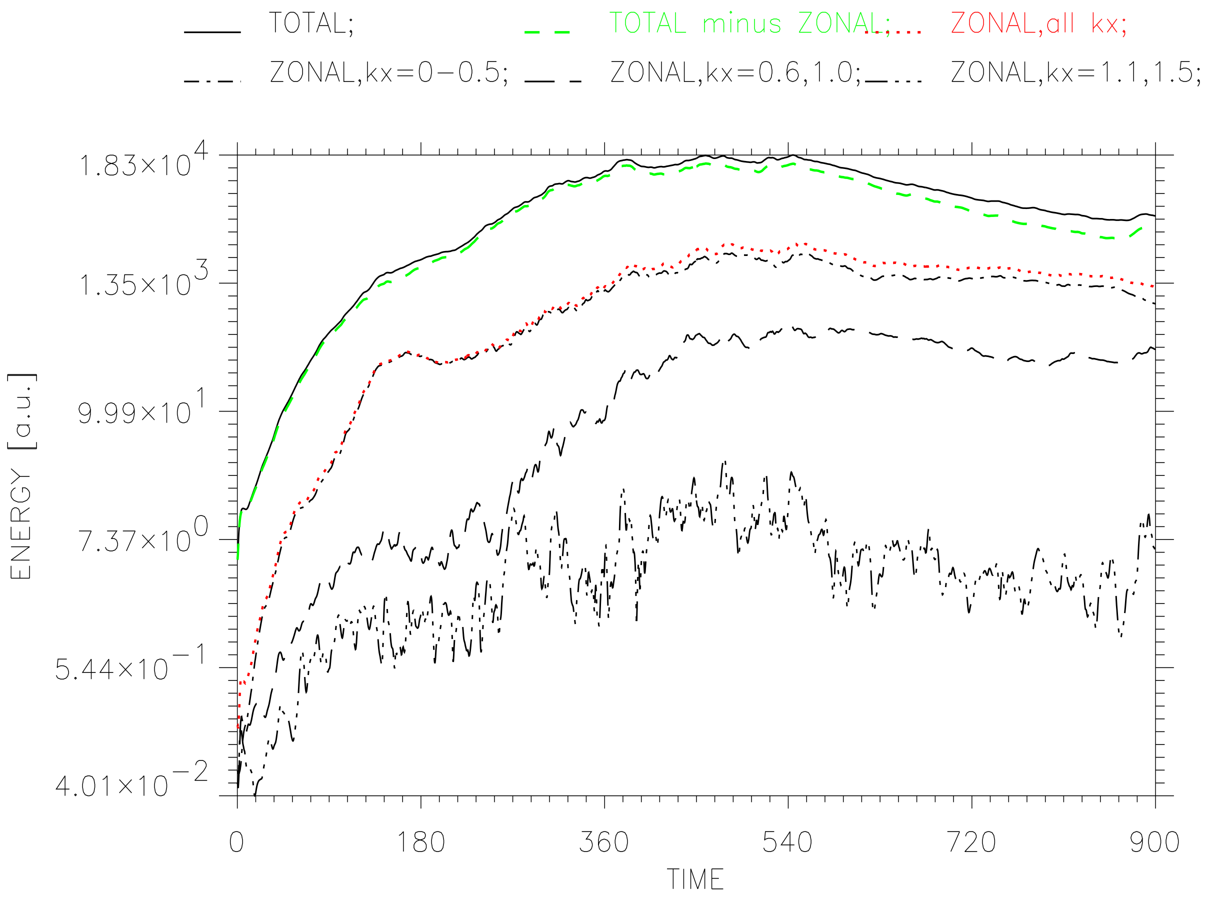


Zonal modes are nonlinearly excited

Total energy: $E_{tot} = \sum_{\mathbf{k}} E(\mathbf{k})$

Zonal energy: $E_z = \sum_{k_x} E(k_x, k_y = 0)$

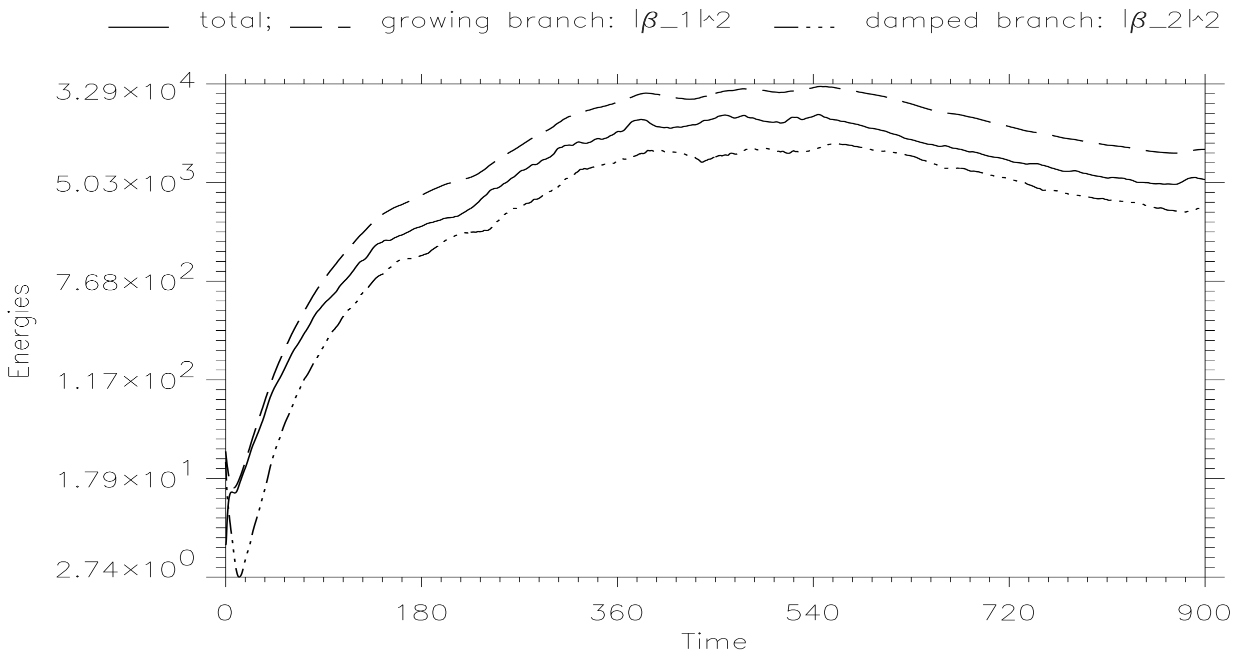
Turbulent energy: $E_t = E_{tot} - E_z$



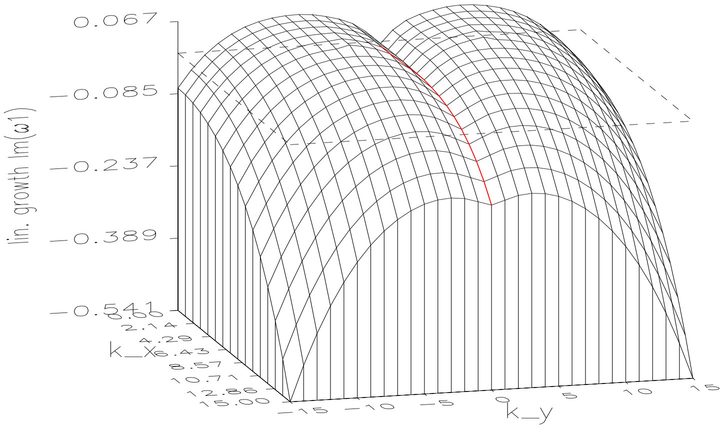
Damped branch $|\beta_2|^2$ nonlinearly excited to finite amplitude

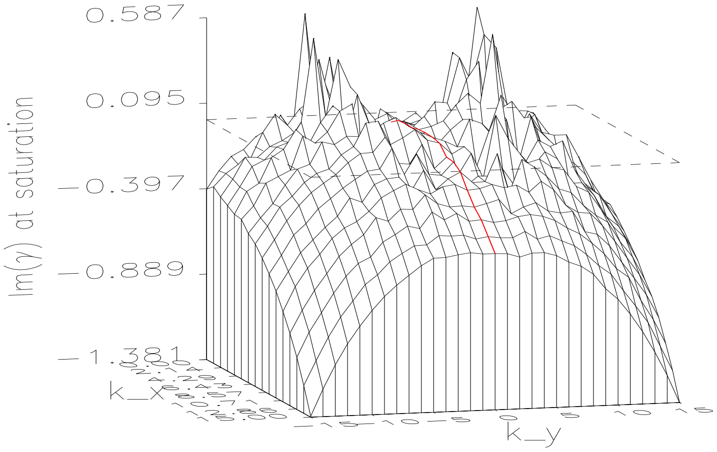
Decomposing the nonlinear solution into the basis set of linear eigenmodes

$$\begin{pmatrix} n(\mathbf{k}) \\ \phi(\mathbf{k}) \end{pmatrix} = \beta_1 \begin{pmatrix} n_1 \\ \phi_1 \end{pmatrix} + \beta_2 \begin{pmatrix} n_2 \\ \phi_2 \end{pmatrix}$$



Zonal modes are nonlinearly damped

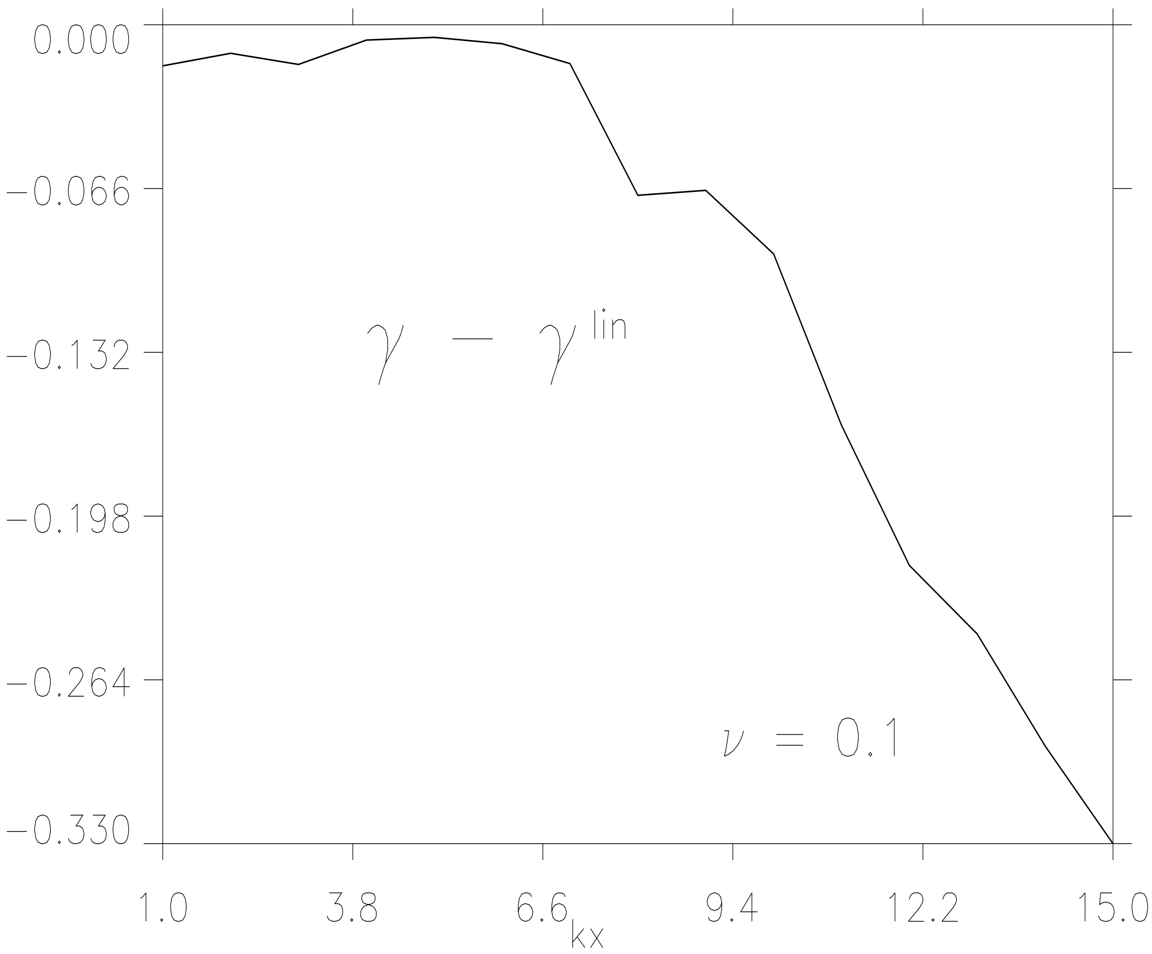




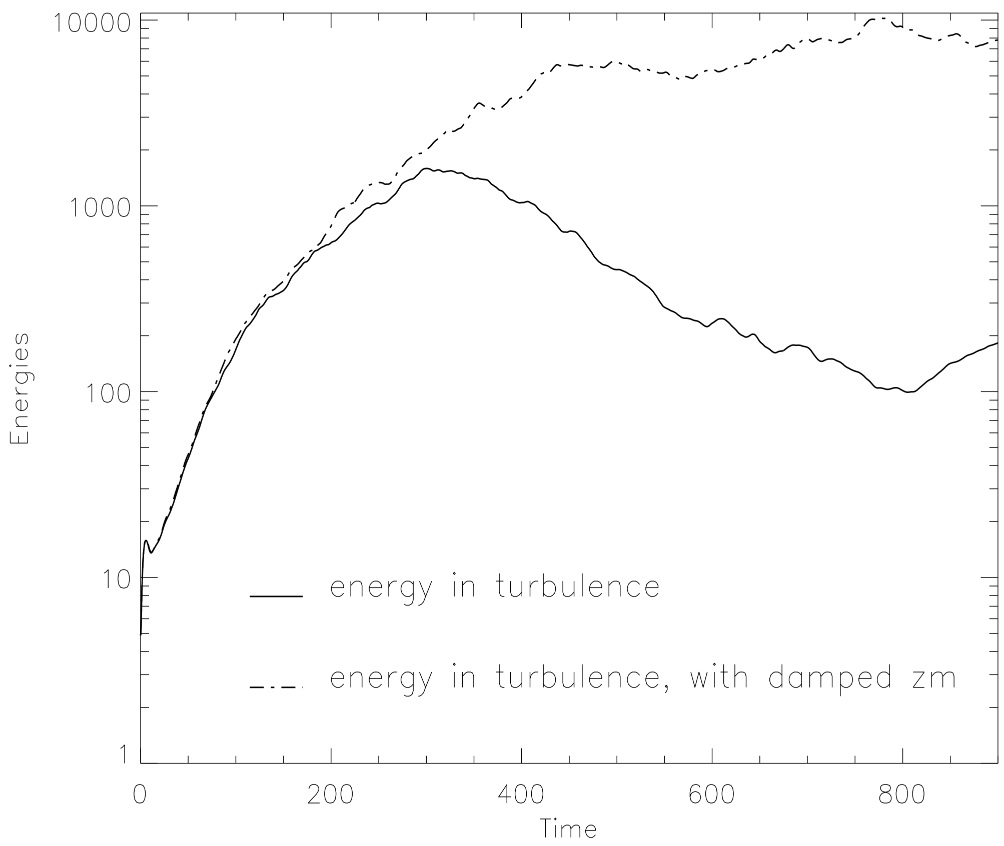
For $\varepsilon^{1/2} \ll 1$, $k_x \ll 1$:

$$\gamma^{\rm z}_{k_x} \simeq -\frac{\nu}{\varepsilon^{1/2}} \left| \frac{\beta^{\rm z}_2(k_x)}{\beta^{\rm z}_1(k_x)} \right|^2$$

Zonal modes ($k_y = 0$) are nonlinearly damped at all k_x.



Energy in turbulence saturates at higher levels when zonal modes are artificially suppressed (by a factor 10) because nonlinear damping of zonal modes is an energy sink for the turbulence



NONLINEAR DAMPING VS SHEARING

$$\frac{dE^t}{dt} = \sum_{k_x}\sum_{k_y \neq 0}\left(\overbrace{2\gamma^t E^t}^{lin.\ drive\ >0} + T_t^{tt} + \overbrace{T_t^{tz}(n^t \phi^z n^t)}^{zonal\ shearing} + T_t^{tz}(n^t \phi^t n^z) \right)$$

$$\frac{dE^z}{dt} = \sum_{k_x}\left(\overbrace{2\gamma^z E^z}^{zonal\ nonlin.\ damping<0} + T_z^{tz} \right)$$

- **Preliminary simulation results:**

$$2\gamma^t E^t \simeq 2\gamma^z E^z \gg T_t^{tz}(n^t \phi^z n^t)$$

- **Analytic theory gives same results**

In CTEM, straining turbulent eddies by zonal flow $\ll$ turbulence suppression by finite-amplitude-induced (nonlinear) damping of zonal modes

CONCLUSIONS

- In CTEM, $\boldsymbol{\nabla}\phi\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}$ NL term, both *drives zonal modes to finite amplitudes* and contributes to the *saturation of the turbulence via a nonlinear damping of zonal modes*
- In CTEM, *saturation of turbulence due to the nonlinear damping of zonal modes is* $\gg$ *that shearing contribution*
- Nonlinear damping of zonal mode is *relevant to other plasma instabilities (ITG)*, as well as to *quasi-geostrophic turbulence* (excitation of zonal flows in the Earth's stratosphere)