Dynamical Models of Happiness

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Abstract: *A sequence of models for the time evolution of one's happiness in response to external events is described. These models with added nonlinearities can produce stable oscillations and chaos even without external events. Potential implications for psychotherapy and a personal approach to life are discussed*

Key Words**:** happiness; hedonics; differential equations; chaos; van der Pol equation

INTRODUCTION

For many people, the pursuit of happiness is a dominant goal of life, and countless books have been written in an attempt to help readers achieve this goal. However, happiness is almost certainly a dynamic process, and it would be a dull and lifeless person indeed who did not experience ups and downs. The power of mathematics has rarely been applied to the dynamics of happiness, although some dynamical models of love, which is certainly one source of happiness, have been inspired by the seminal musings of Strogatz (1988, 1994). Essentially the same model was described earlier by Rapoport (1960), and it has also been studied by Radzicki (1993). Related discrete dynamical models of the verbal interaction of married couples have recently been advanced by Gottman, et al. (2002). Although Strogatz's model was originally intended more to motivate students than as a serious description of love affairs, it makes several interesting and plausible predictions and suggests extensions that produce an even wider range of behavior. This paper is written in the same spirit and extends the ideas to the temporal response of one's happiness to external events, which in the presence of nonlinearities can exhibit oscillations and chaos even without external

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events. It concludes with some implications for psychotherapy and for a personal approach to life.

One might suppose that it is essential from the outset to define happiness, which embodies many emotions, including excitement, pleasure, joy, contentment, serenity, fulfillment, and satisfaction, among others (Orsucci, 2001). Some people (e.g., Goleman, 2003) distinguish between mood, which is not triggered by outside events, and emotions, which are, and advance the Buddhist philosophy that true happiness is a mood not related to external circumstances. However, the model presented here is not sensitive to the precise definition of happiness, and most people can readily decide whether they are happy at the moment and how happy or unhappy they were at various stages of their lives, although there is a preferential tendency to report positive happiness, perhaps as a result of peer pressure (Diener & Diener 1996, Lykken & Tellegen 1996, Diener, et al. 1999). Of course happiness is not a simple scalar quantity, since one may be happy with one's job but not with one's marriage, for example (Larson, et al.*,* 2001), or one can be unhappy about the past but hopeful and upbeat about the future (Seligman, 2002). For the purpose of this paper, it will suffice to take some weighted average on the various feelings that go into the sense of happiness or to apply the model to each component separately. It is essential to understand that the happiness described here is the subjective feeling of the individual rather than the judgment of someone else, however objective.

SIMPLE LINEAR MODEL

In a companion paper (Sprott, 2004), it was shown that two individuals (Romeo and Juliet), even in the simplest linear case, can exhibit temporal oscillations in their love for one another, so that Romeo's love R for Juliet is given by

$$\frac{d^2R}{dt^2} + \beta R + \omega^2 R = 0 \tag{1}$$

where β is the damping (rate of decay) and ω is the natural resonant frequency of oscillation (in units of radians per unit time). Without loss of generality, the frequency can be taken as $\omega = 1$, which amounts to measuring time in units of $1/\omega$. Equation 1 is the familiar expression for a damped (if $\beta > 0$) harmonic oscillator describing many phenomena such as a mass on a spring, a small-amplitude pendulum, or a resonant electrical circuit. The feedback that allows oscillations is through

Juliet's reactions to Romeo's love. This is the simplest dynamical model for Romeo's love and his resulting happiness.

Many things can influence happiness besides love, and there are many feedback loops, most of which are internal to Romeo and do not require a Juliet. Thus it is reasonable to propose Eq. 1 as the simplest general dynamical model of happiness. However, love and happiness are not identical. Indeed, one usually is happiest (at least in the sense of feeling the most excited) when falling in love and saddest when that love is lost. There is a tendency to acclimate to whatever good things life provides, whether it be love, money, friends, health, education, or fame. People who have such assets tend on average to be no happier than those who have acclimated to being alone, poor, sick, or uneducated. In fact, depression is primarily a disease of the affluent (Seligman, 2002). Prisoners are likely to be unhappy when first incarcerated, but eventually they probably become as happy as their guards on average. While there is ample evidence that people with few friends are less healthy and tend to die younger (House, et al., 1988), it does not follow that their subjective feeling of happiness is any less than those with many friends, since they are likely to acclimate to their loneliness. The same is true of people with positive emotions early in life (Danner et al*.,* 2001). Seligman (2002) calls this tendency to acclimate to one's conditions the "hedonic treadmill."

Thus it is more reasonable to associate happiness with the rate of change of R, or $H = dR/dt$. Conversely, R is the integral of H and is what others presume your happiness to be based on your circumstances as opposed to H, which represents your true feelings. Note that with positive damping ($\beta > 0$), H will average to zero over a sufficiently long time. Indeed, a sample of 3050 American adults report feeling happy on average 54% of the time and unhappy or neutral 46% of the time (Fordyce, 1988). Although happiness is assumed to average to zero, that does not mean that an individual is happy only half the time since long periods of happiness can be interrupted by short periods of intense unhappiness such as might accompany the loss of a loved one or other tragic event. Dying suddenly before one's spouse does while still in excellent health is another way to beat the average.

The many external events or circumstances that effect one's happiness constitute a time-dependent forcing function $F(t)$, so that the general happiness model is

$$\frac{d^2R}{dt^2} + \beta\frac{dR}{dt} + R = F(t) \tag{2}$$

This formula differs from the non-dynamical model proposed by Seligman (2002), which in terms of the current variables would read $H = dR/dt = F(t) + S + V$, where S is a set range that we inherit and V represents factors that we can voluntarily control. The only equilibrium for Eq. 2 occurs when $F(t)$ is a constant, and it has $H = 0$ and $R = F$. Therefore happiness (and sadness) are transient phenomena, and the only way to find lasting happiness in this model is if dF/dt is always positive, corresponding to an ever increasing stimulus (the "hedonic treadmill").

Note that a second-order ordinary differential equation with a time-dependent forcing term such as Eq. 2 (a nonautonomous equation) can be reduced to three first-order (autonomous) differential equations that do not involve time explicitly by defining two new variables, one equal to dR/dt (or H) and the other equal to the argument of $F(t)$ whose time derivative is unity. Driven oscillators ($F(t) \neq 0$) have a three-dimensional state space and thus can exhibit chaos in the presence of a nonlinearity, as will be shown.

WINNING THE LOTTERY

Consider a single unexpected event such as winning the lottery or death of a spouse that perturbs a person at equilibrium. Depending on the value of β, the person's response $R(t)$ can take one of the forms in Fig. 1. On the left are cases with negative damping ($\beta < 0$), and on the right are cases with positive damping ($\beta > 0$). The upper cases are overdamped ($\beta > 2\omega$), and the lower cases are underdamped ($\beta < 2\omega$). Critical damping occurs for $\beta = 2\omega$, and the response for this case is shown in Fig. 2. Critical damping is the condition for which $R(t)$ returns to zero as rapidly as possible without overshooting. Shock absorbers are designed to be critically damped so as to minimize the shock when a car hits a bump without continuing to oscillate afterward, and it may be that emotionally healthy people have the same characteristic. Note that after the event there is an initial euphoria ($H > 0$) that quickly fades and is replaced by a period of mild unhappiness ($H < 0$) until equilibrium is slowly restored. Such effects have been documented in lottery winners and accident victims (Brinkman, et al. 1978).
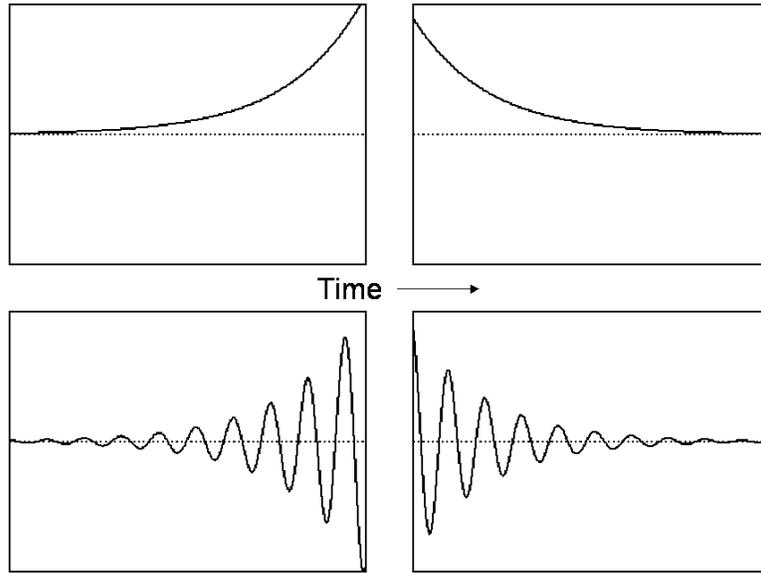
Fig. 1. Possible responses of the happiness model in Eq. 1 to an isolated event.
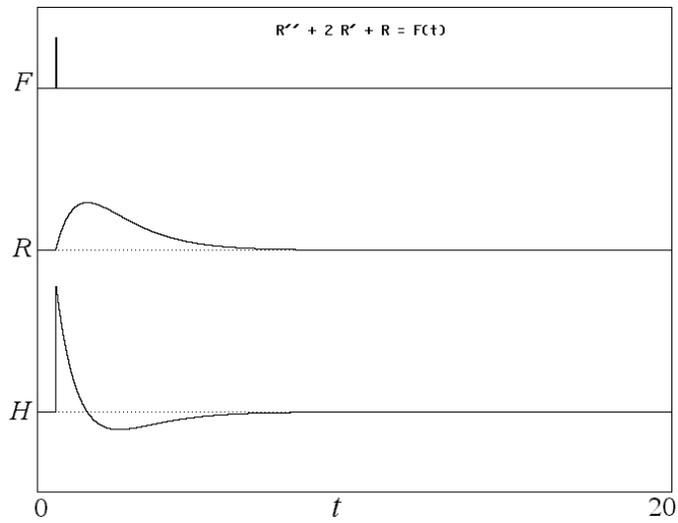

Fig. 2. Response of happiness to a single event like winning the lottery from Eq. 1.

DRUG OR OTHER ADDICTION

A periodic stimulus such as taking an antidepressant drug at regular intervals with critical damping gives the response in Fig. 3. Happiness is improved for the first few doses, but then the subject habituates or grows tolerant to the drug, and its effect averages to zero. When the medication ceases, there is an unhappy period of withdrawal until the subject returns to equilibrium. To have a lasting beneficial effect, the dosage must be continually increased, leading to addiction (or dependence). When the stimulus is socially valued (such as money), the addiction is called is called "ambition" (or "greed" in extreme cases).
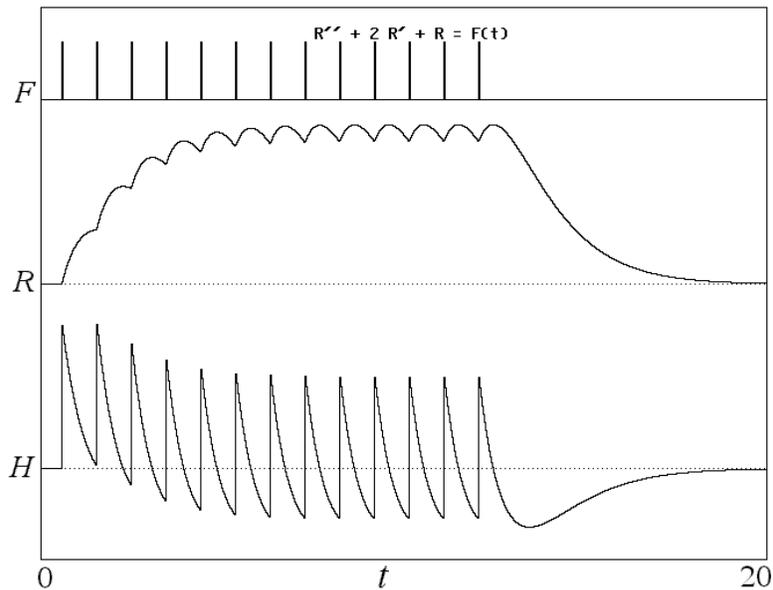

Fig. 3. Response of happiness to a periodic stimulus such as drug adiction from Eq. 1.

REAL LIFE

In real life, things are happening all the time, and the dynamics probably look more like Fig. 4 in which $F(t)$ is Gaussian white noise with mean zero (good and bad events are equally likely) and the system is critically damped. Note that both $R(t)$ and $H(t)$ average to zero but that the happiness one feels is much more volatile than that perceived by

others and they tend to be out of phase, with the perception (*R*) lagging the reality (*H*). The tendency of happiness to revert to a set level has been reported by Lykken & Tellegen (1996) and many others. In the model presented here, the set level is zero.
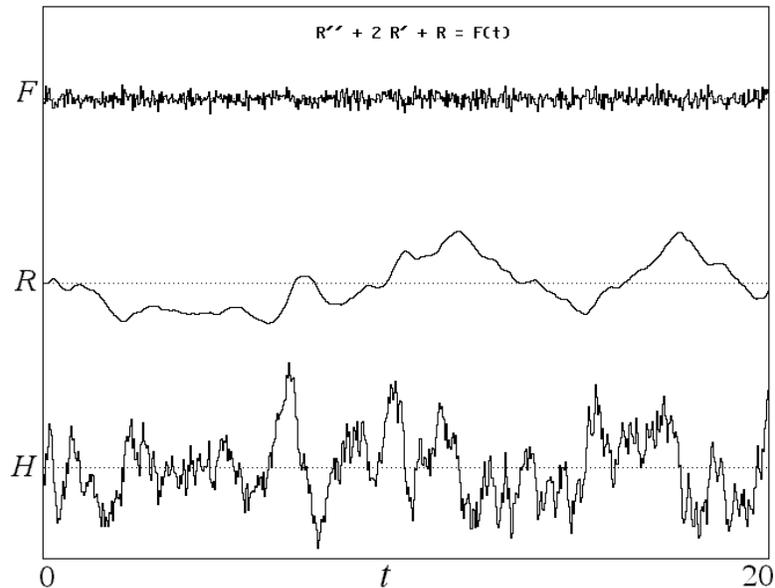

Fig. 4. Response of happiness to Gaussian white noise typical of real life from Eq. 1.

NONLINEAR EFFECTS

People respond to events with values of *β* and *ω* that characterize their psychological profile. Attempts to categorize people by personality models using a small number of variables have a long history (Gray, 1999), including such measures as the Five Factor Models of Goldberg and of Costa and McCrae, the Myers-Briggs Type Indicator, Cattell's 16PF (sixteen-personality-factor) Taxonomy, and Eysenck's Three-Factor Model. Some people have small damping and respond to events with oscillations that only slowly damp or even grow for *β* < 0. Such behavior might correspond to a bipolar disorder (manic depression), and many people experience an emotional roller coaster (Solomon & Corbit 1974), although such oscillations tend to decrease as one ages (Mroczek,

& Kolarz, 1998). Oscillations would grow until some nonlinearity intervenes. For example, the person might take an antidepressant medication when depression reaches some threshold. Such threshold behavior has been proposed in a model that exhibits cyclic behavior and hysteresis as a result of the competition of two unstable equilibria (Guastello, 1984).

A simpler example of a nonlinearity that does not involve a bifurcation or hysteresis would be to replace the β in Eq. 2 with $\beta(1 - R^2)$, which amounts to measuring R in units such that $R = 1$ corresponds to the value at which the negative damping becomes positive. Qualitatively similar results follow from the function $\beta(1 - |R|)$. The resulting system

$$\frac{d^2R}{dt^2} + \beta(1 - R^2)\frac{dR}{dt} + R = F(t) \tag{3}$$

is identical to the driven van der Pol equation, first used to model oscillations in vacuum tube electrical circuits (van der Pol, 1926). It has stable limit-cycle solutions as shown in Fig. 5 for $\beta = -3\omega$ even for $F(t) = 0$. This example shows a person whose equilibrium is periodically punctuated by brief intervals of alternating extreme happiness and sadness. Such limit cycles have been observed in models of the love dynamics of the 14th century Italian poet Francis Petrarch (1304-1374) and his beautiful married platonic mistress Laura (Rinaldi 1998). For periodic forcing, the system as shown in Fig. 6 with $\beta = -3\omega$ and $F(t) = 5\sin 1.788t$ is maximally chaotic with Lyapunov exponents of 0.1932, 0, and –1.8789 and a Kaplan-Yorke dimension of 2.1028 (Sprott 2003).

ANTICIPATION

Sometimes happiness involves the anticipation of events yet to happen or the surprise of things improving faster than expected. For example, one can acclimate to getting a salary raise of 10% every year, while a 15% raise gives pleasure and a 5% raise causes unhappiness. Such effects can be included in a more general model that includes a third derivative (a "jerk")

$$\frac{d^3R}{dt^3} + \alpha\frac{d^2R}{dt^2} + \beta(1 - R^2)\frac{dR}{dt} + R = 0 \tag{4}$$

that can produce chaos even without an external $F(t)$ (Sprott, 1997). A new damping parameter α has been introduced to make the model completely general. Integrating this equation shows that it is a nonlinear oscillator driven by a memory term that involves the integral of R.
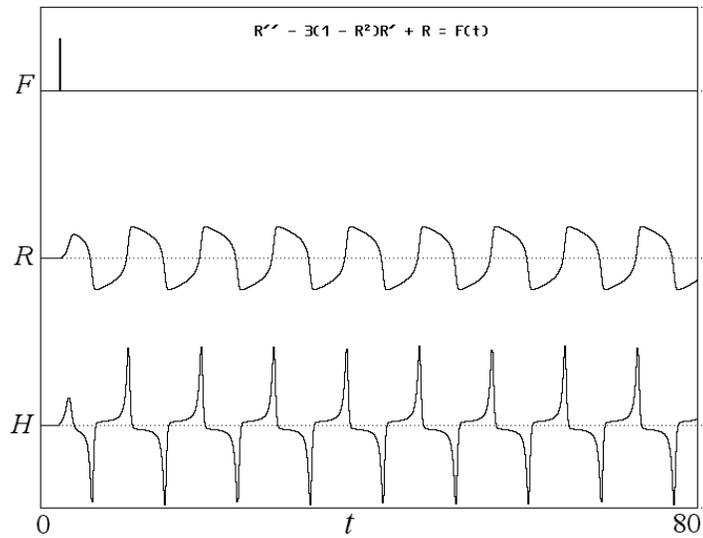
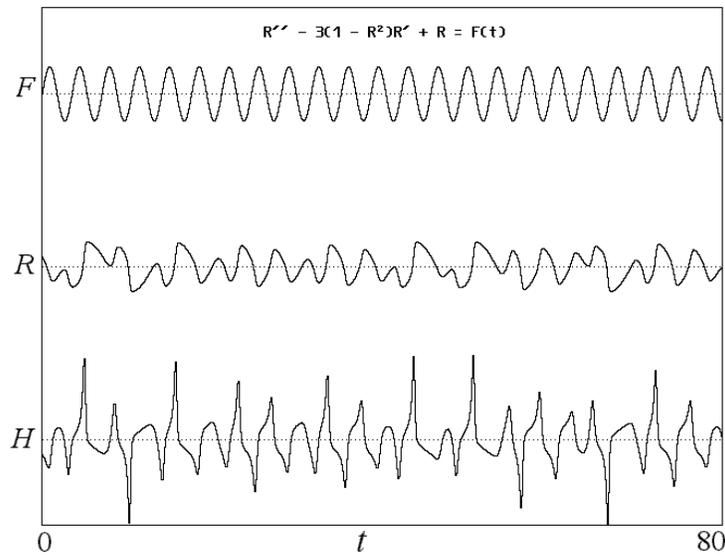**Fig. 5.** Limit cycle behavior modeling a bipolar disorder from Eq. 2.

Fig. 6. Chaotic behavior of happiness with a periodic forcing from Eq. 2.
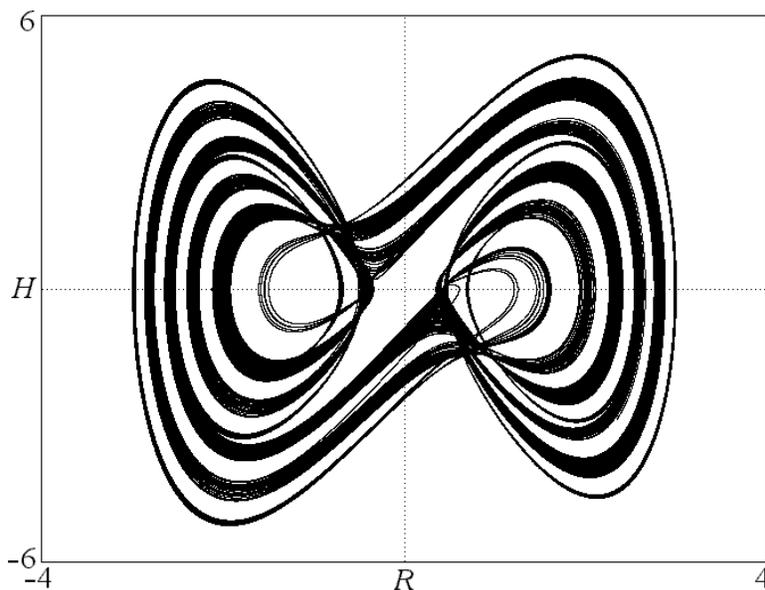
Fig. 7. Strange attractor from the happiness model with anticipation in Eq. 2.

Equation 4 generalizes Eq. 2 much the same as Eq. 2 generalizes Seligman's (2002) non-dynamical model. Figure 7 shows one strange attractor that arises from Eq. 4 with $\alpha = 0.5$ and $\beta = -5.6$, for which the Lyapunov exponents are 0.0570, 0, and –0.5570, and the Kaplan-Yorke dimension is 2.1023.

TESTS OF THE MODEL

The models are amenable to experimental test because they involve single individuals subjected to external stimuli. The simplest test of the model would involve recording the self-reported happiness or unhappiness of isolated individuals in the aftermath of an unexpected event, either good or bad. Examples include winning the lottery, death of a loved one, or unanticipated incarceration, where a response similar to Fig. 1 is expected, from which the parameters β and ω can be determined. One could also simultaneously record the perception of someone close to the subject such as a spouse, whose opinion should be roughly the integral of the individual's self-reported happiness.

Another test would be to survey many individuals from diverse segments of society, asking how happy or unhappy they are at that instant and whether they have been predominantly happy or unhappy over the course of their lives. In such a study, one should discount those individuals whose status has recently changed, for better or for worse. Some such studies have been done for lottery winners and accident victims (Brickman, Coates & Janoff-Bulman 1978), those whose spouse or child died (Lehman, et al*.,* 1987), family caregivers of Alzheimer's sufferers (Vitaliano, et al*.,* 1991), religious people and freethinkers (Lykken 2000), severely ill cancer patients (Breevelt & van Dam, 1991), quadraplegics (Gerhart, Koziol-McLain, Lowenstein, & Whiteneck, 1994), and those with severe disabilities (Diener & Diener 1996), spinal cord injuries (Silver, 1992), and mental problems (Delespaul & De Vries, 1987). Suh, Diener, and Fujita (1996) report that good and bad events have almost no effect on happiness after six months.

Note that the model should also apply to any quality to which an individual acclimates, including wealth, health, beauty, intelligence, and spirituality, as well as the physical senses of taste, touch, smell, hearing, and sight. Thus it could be tested in a wide variety of contexts.

SUMMARY AND CONCLUSIONS

Simple linear models of happiness can produce surprisingly complex dynamics, much of which rings true to common experience. Even simple nonlinearities can produce chaos when there are periodic external events or when there is anticipation of things to come. The existence of chaos in dynamical models of happiness implies a degree of unpredictability, but it also suggests the possibility of stabilizing one's emotions with only small perturbations to the system or synchronizing one's feelings to another dynamical system such as the weather or the economy. Two or more chaotic individuals who interact might find their moods in synchrony. These models are gross simplifications since they assume that happiness is a simple scalar variable and that individuals respond in a consistent and mechanical way only to that variable. Some implications of the models include the following:

1. The simplest model supposes that over a sufficiently long time, people tend to acclimate to their circumstances and thus experience equal amounts of happiness and unhappiness. Therefore, constant happiness is an unrealistic and unobtainable goal.

2. Suicide is an irrational response to unhappiness, similar to bailing out of the stock market at the bottom. The model predicts that if you wait long enough, happiness will return, even if only by virtue of acclimating whatever is causing the unhappiness.

3. Others tend to perceive less volatility in you than you feel since they observe primarily your responses to external events (R) rather than your true feelings (H), and therefore they often wrongly conclude how happy you are.

4. Since individuals will tend to acclimate to their circumstances, long prison terms may be ineffective if the goal is punishment rather than deterrence or protection of society.

5. Individuals may be characterized by two parameters β (how rapidly they return to equilibrium after a perturbing event) and ω (the frequency with which their feelings change). These parameters may be of diagnostic use in describing psychological health.

6. Healthy individuals may have $\beta \sim 2\omega$, corresponding to critical damping, while people with $\beta < 0$ exhibit bipolar behavior. A strongly overdamped person ($\beta \gg 2\omega$) may be largely devoid of emotions and unaffected by events, whether good or bad.

7. A reasonable goal of psychotherapy might be to alter the parameters β and ω so that the patient responds to external events in a more healthy manner. It is an open question the extent to which such therapy is effective, since the parameters may be largely fixed by one's personality.

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